Lecture 5: Decision Trees

Decision Trees

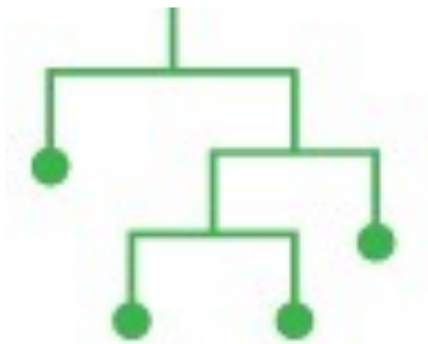
- **Decision Trees**
  - extract a set of classification rules to classify a given instance
    - like IF-ELSE statements, testing different attributes
  - ML model that is highly interpretable by humans

Decision Trees

	#friends	funnyscore	Likes
Post1	40	8	28
Post2	36	10	28
Post3	20	6	16
Post4	56	4	31
Post5	58	0	29
Post6	46	10	33

```
if #friends >= 50
    if funny_score > 3
        Likes >= 30
    else
        Likes < 30
else
    if funny_score > 9
        Likes >= 30
    else
        Likes < 30
```

Decision Trees



Learning to Play the 20 Questions Game

http://en.akinator.com/


Question N°1
Is your character a female?
Yes
No
Don't know
Probably
Probably not


Correct
Question N°2
Is your character real?
Yes
No
Don't know
Probably
Probably not


Correct
Question N°3
Is your character a famous youtuber?
Yes
No
Don't know
Probably
Probably not


Correct
Question N°4
Is your character older than 35 years old?
Yes
No
Don't know
Probably
Probably not


Correct
Question N°5
Is your character American ?
Yes
No
Don't know
Probably
Probably not


Correct
Question N°6
Is your character an actor?
Yes
No
Don't know
Probably
Probably not


Correct
Question N°7
Is your character the president or was he?
Yes
No
Don't know
Probably
Probably not


Correct
Question N°8
Is your character black?
Yes
No
Don't know
Probably
Probably not


I think of
Barack Obama
Former President of the United States
© Copyright / IP Policy
Yes
No

Decision Trees

- Which attributes of a person to test <u>first</u>, to guess as fast as possible?
  - Is the person a man or a woman?
  - Is the person older than 5 years old?

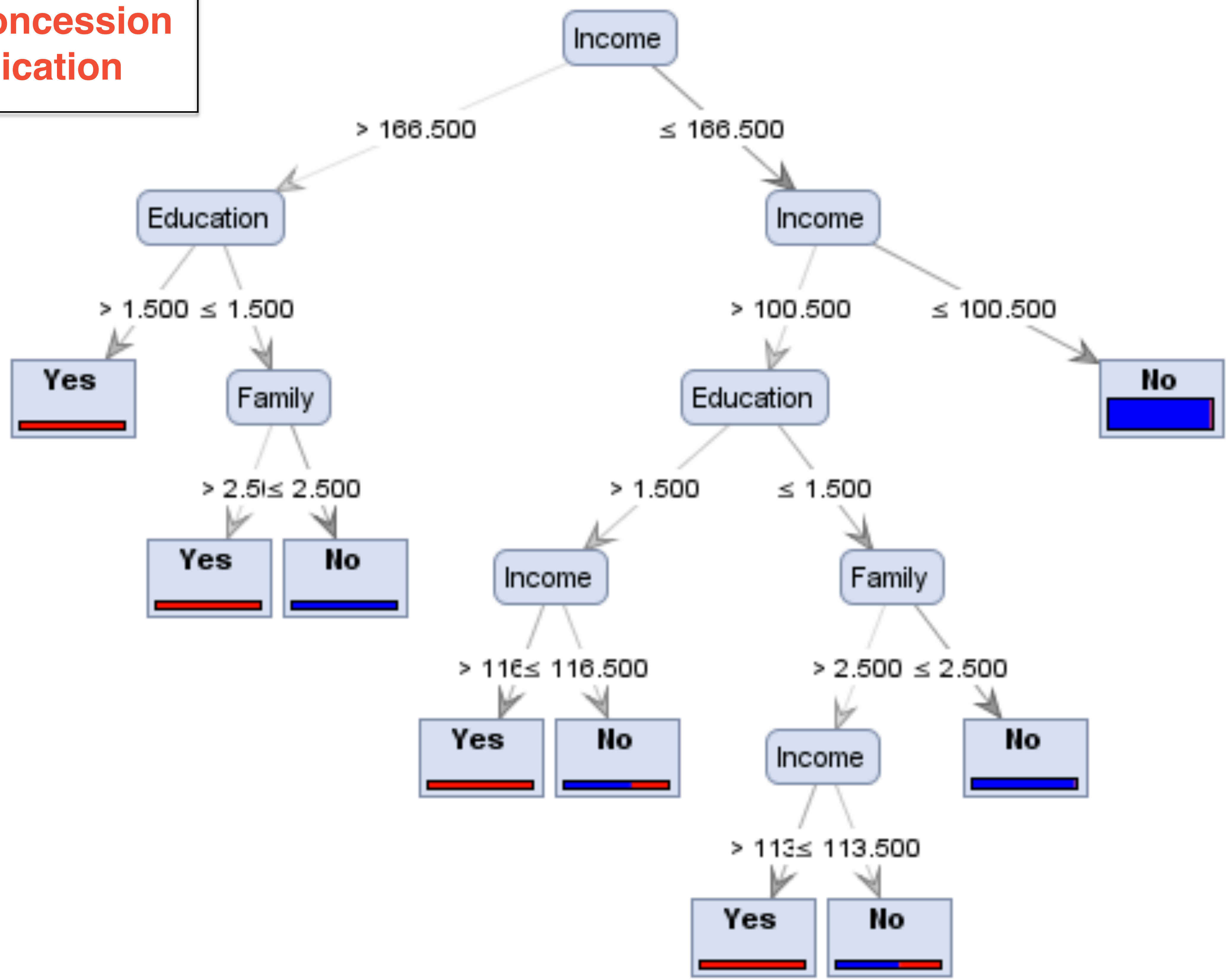
<u>Information Gain</u>

- G("Gender") = 0.9
- G("Lives_in_USA") = 0.73
- G("Is_Politician") = 0.36
- …

Entropy of a set (or dataset)

Decision Trees

Loan Concession Application



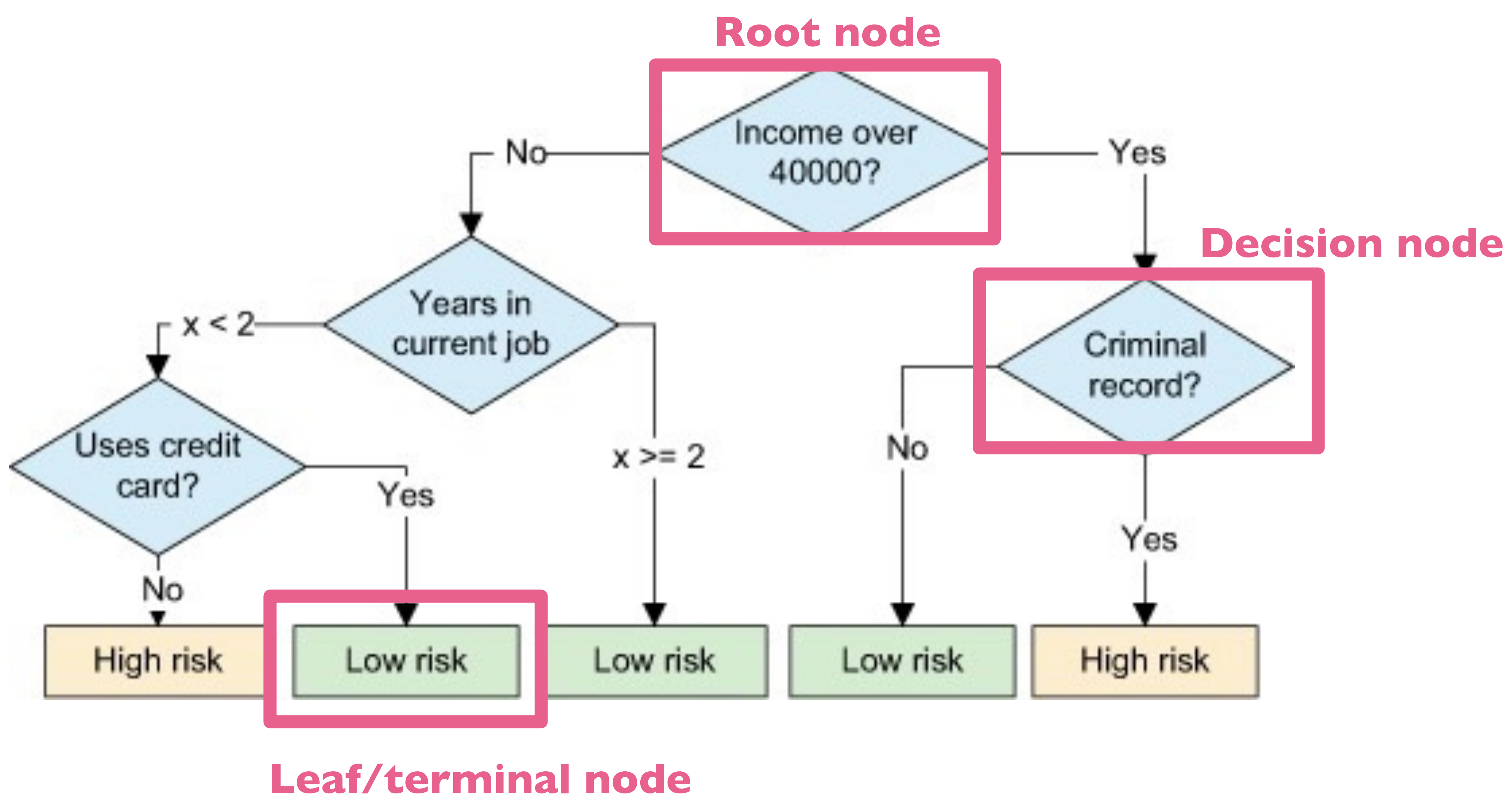
Decision Trees

- **Decision Tree**
  - Simple ML model/algorithm
  - Widely used in practice
  - Easy to *interpret*

- Performs a series of tests on attributes of an instance
  - eventually leading to decision/prediction about the class of that instance

Decision Trees

A decision tree for testing loan suitability

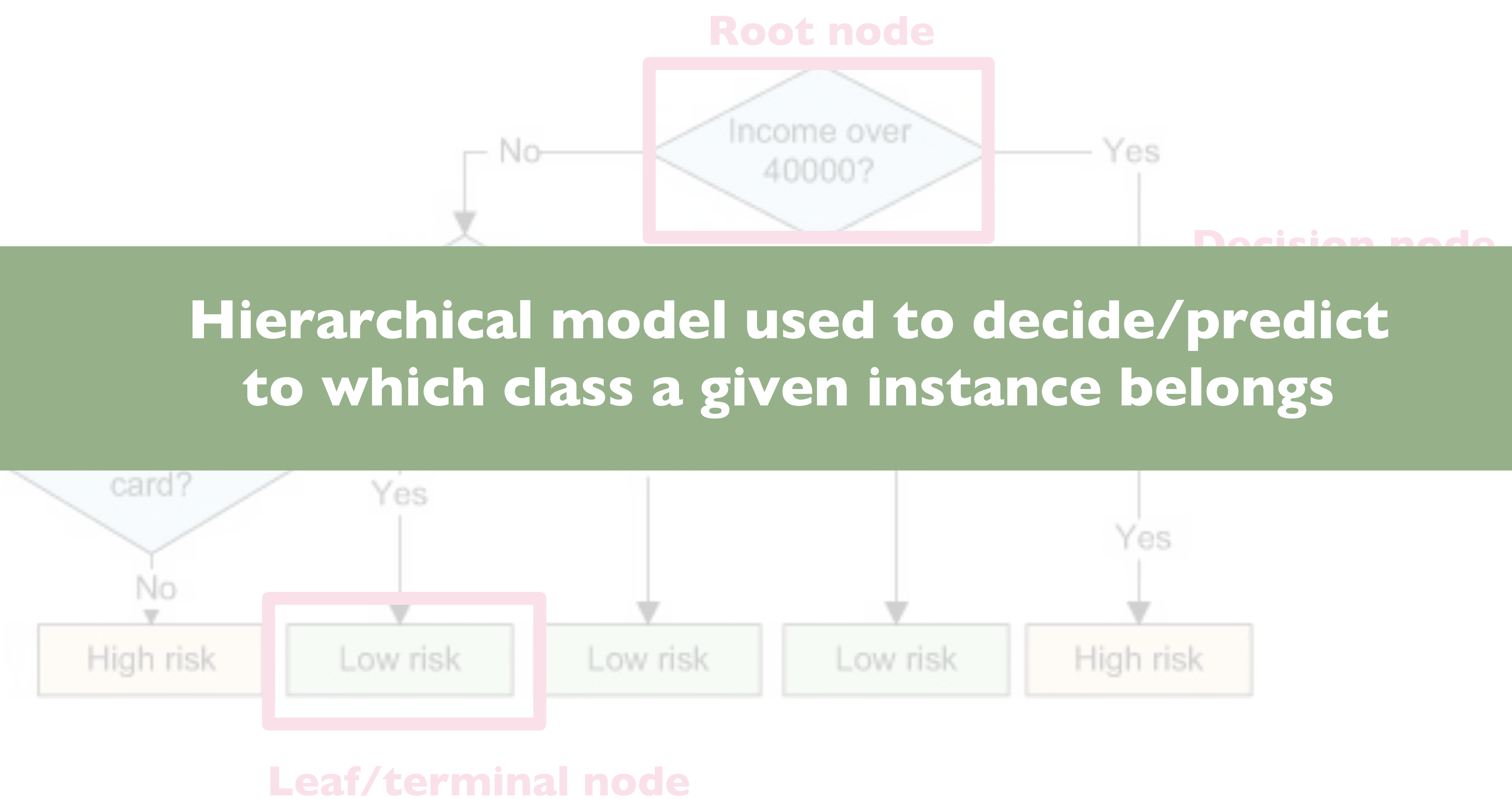


"Real-World Functional Programming with Examples in F# and C#". (Skeet and Petricek, 2010)

Decision Trees

A decision tree for

testing loan suitability



Hierarchical model used to decide/predict to which class a given instance belongs

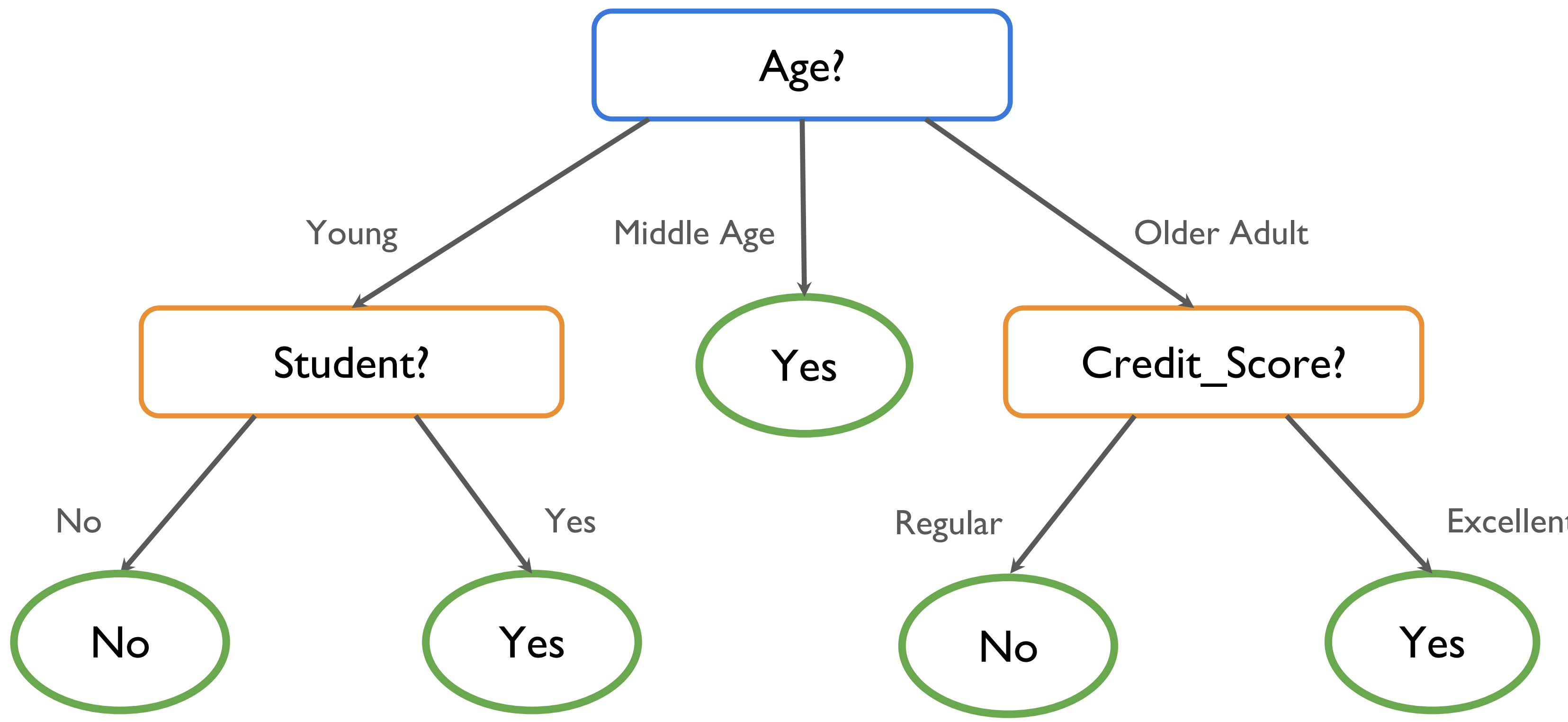
"Real-World Functional Programming with Examples in F# and C#". (Skeet and Petricek, 2010)

Decision Trees

How likely it is that a given client will buy a computer?

Each instance, x_i, is described by the following attributes

$x_i = [\text{Student}, \text{Age}, \text{Credit\_Score}]$, where $y_i \in \{\text{Yes}, \text{No}\}$



- Each non-leaf node tests the value of a given attribute
- Each branch corresponds to one possible value of that attribute
- Each leaf corresponds to predicting one particular class
- The path from the root node to a leaf defines a **classification rule**

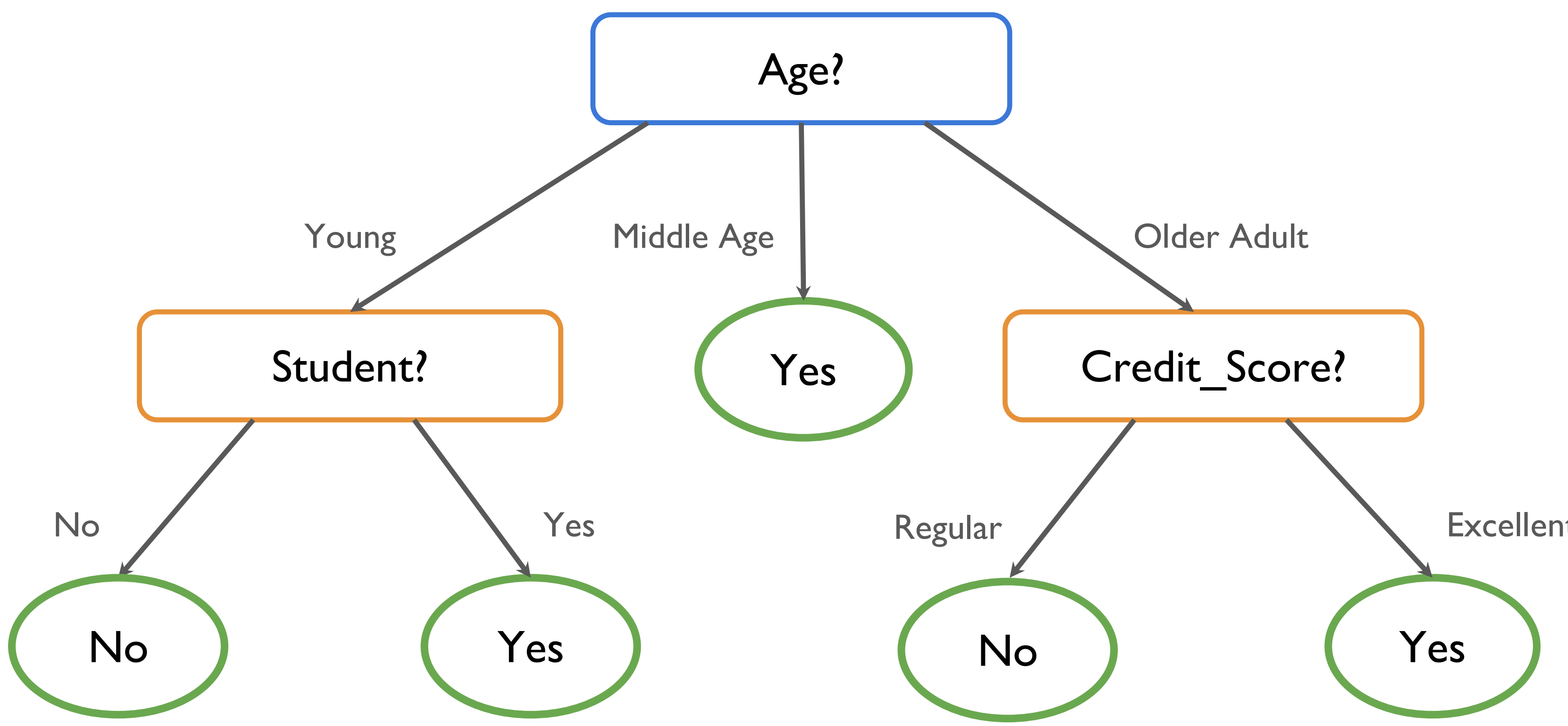
Decision Trees

How likely it is that a given client will buy a computer?

Instance to be classified:

Student	Age	Credit_Score
Yes	Young	Regular

Will_Buy_Computer
??



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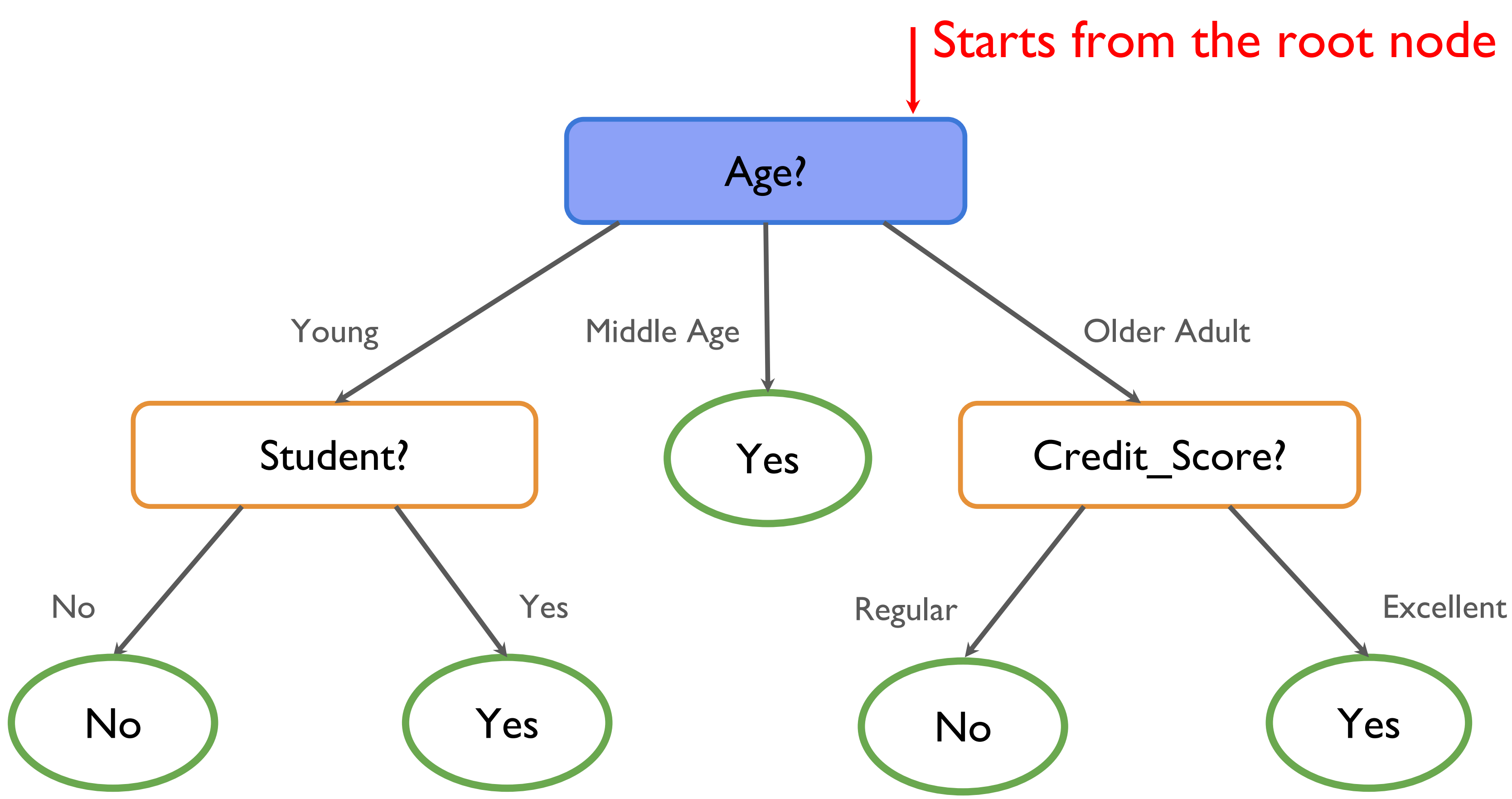

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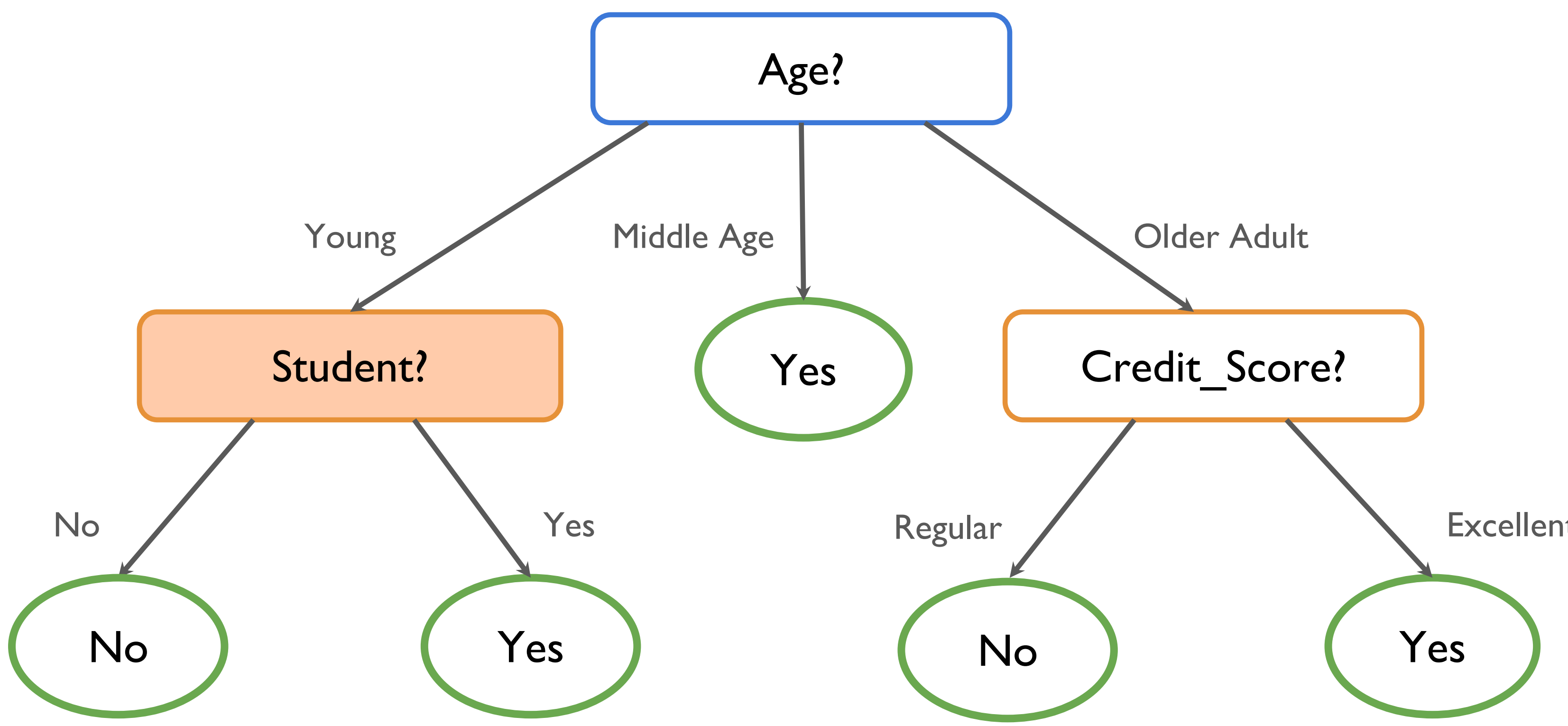

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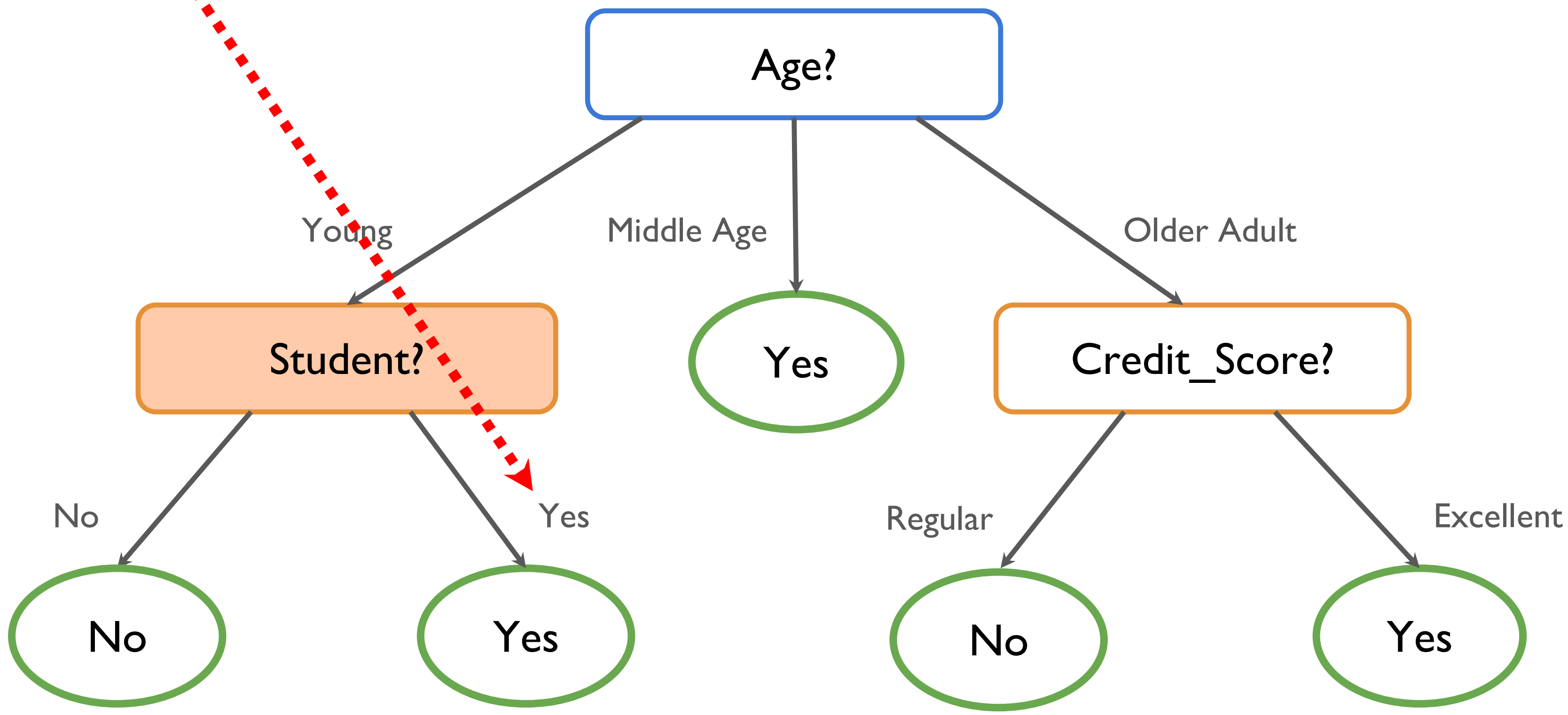

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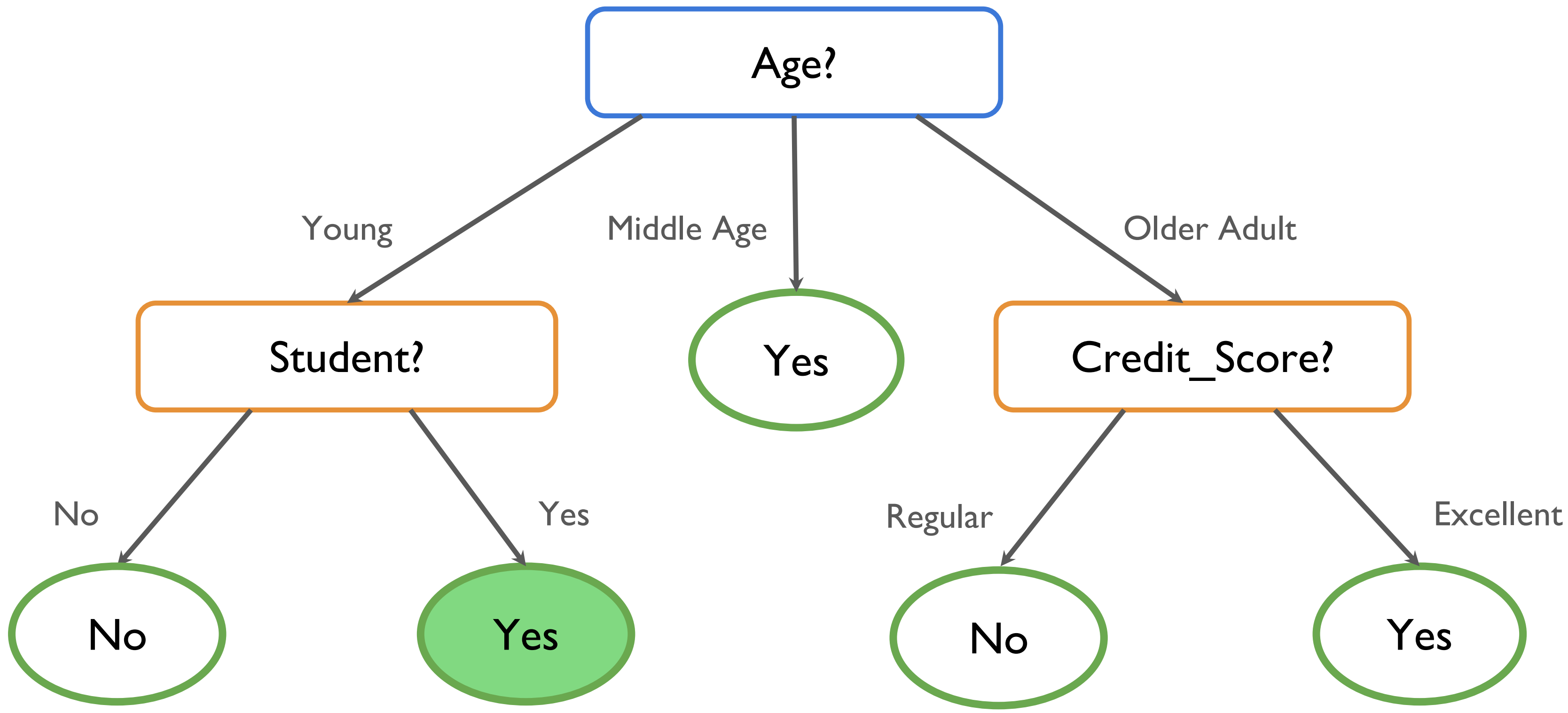


Decision Trees

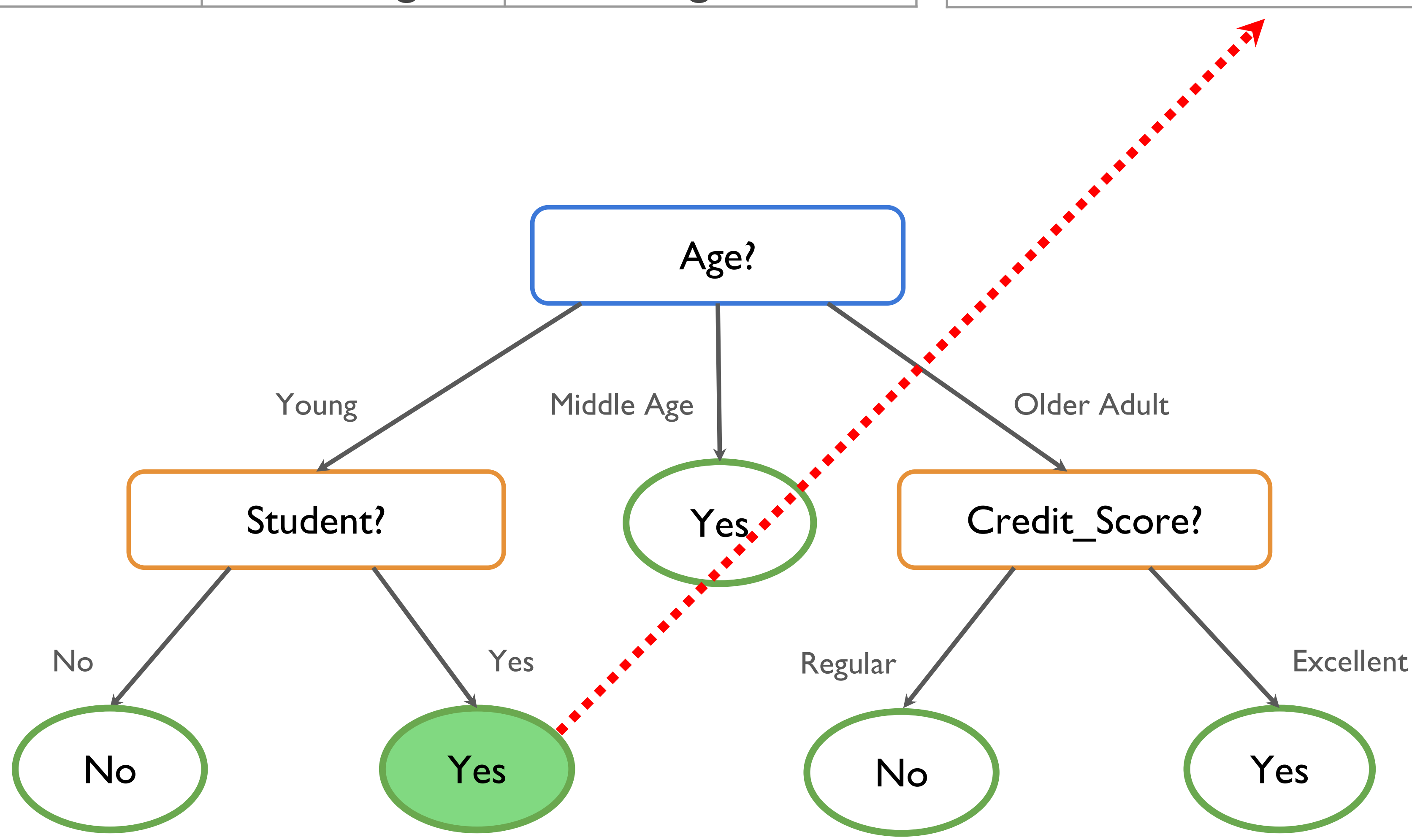
How likely it is that a given client will buy a computer?

Instance to be classified:

Student	Age	Credit_Score
Yes	Young	Regular

Predicted Class

Will_Buy_Computer
Yes



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Decision Trees

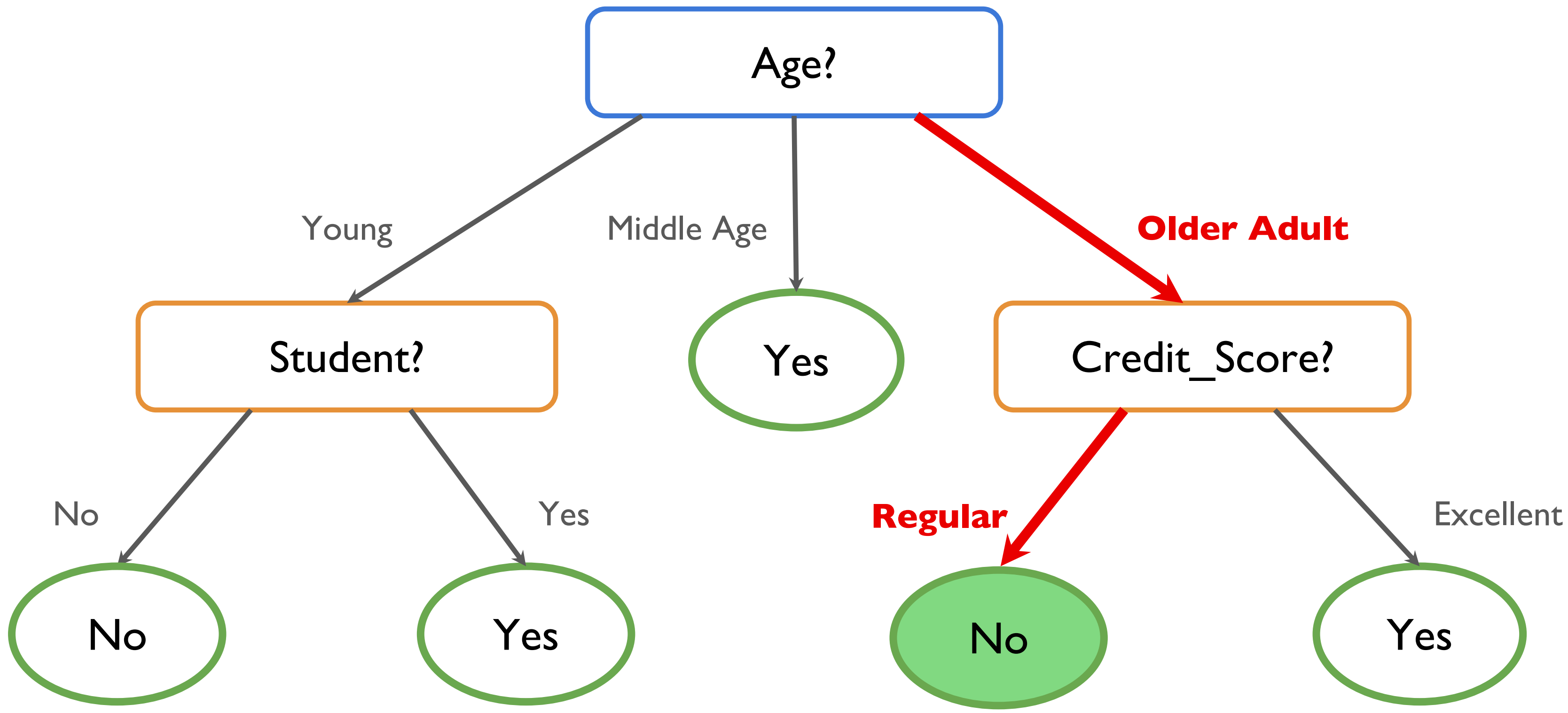
How would the following instance be classified?

Instance to be classified:

Student	Age	Credit_Score
No	Older Adult	Regular

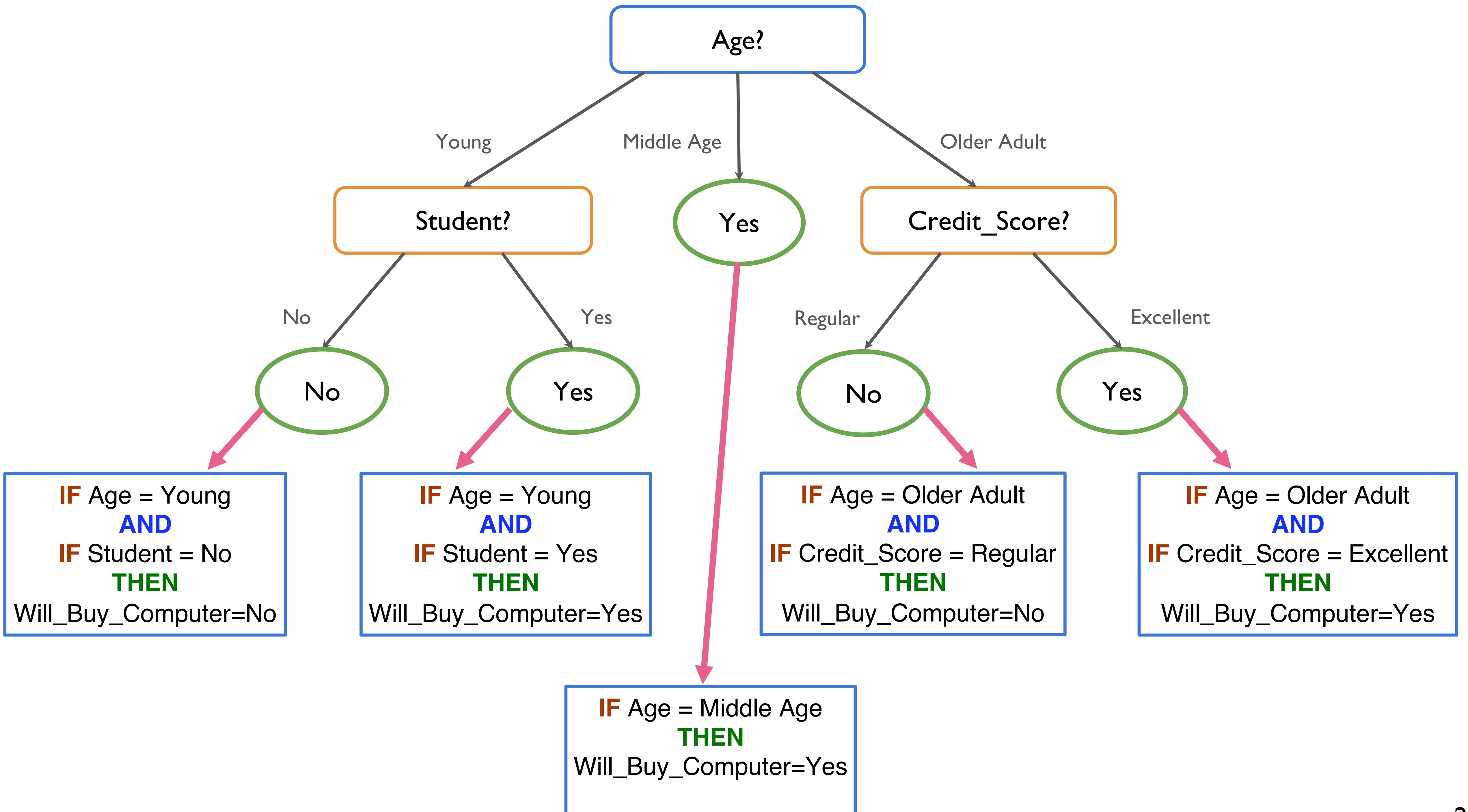
Predicted Class

Will_Buy_Computer
No



Decision Trees

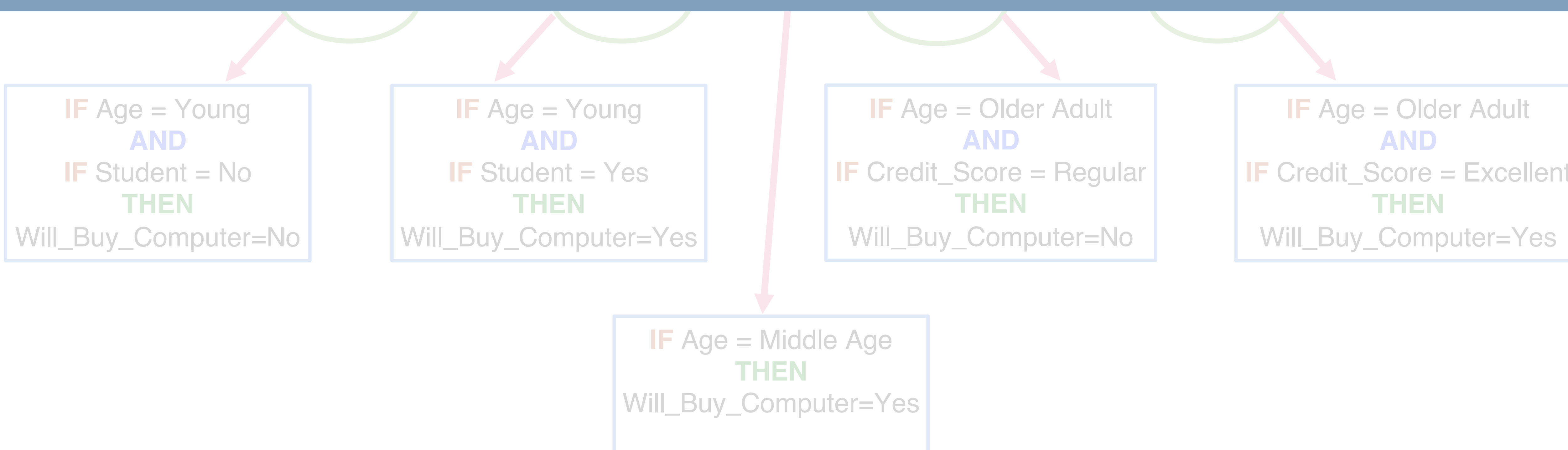
Decision trees encode classification rules via (implicit) IF-ELSE statements



Decision Trees

Decision trees encode classification rules via (implicit) IF-ELSE statements

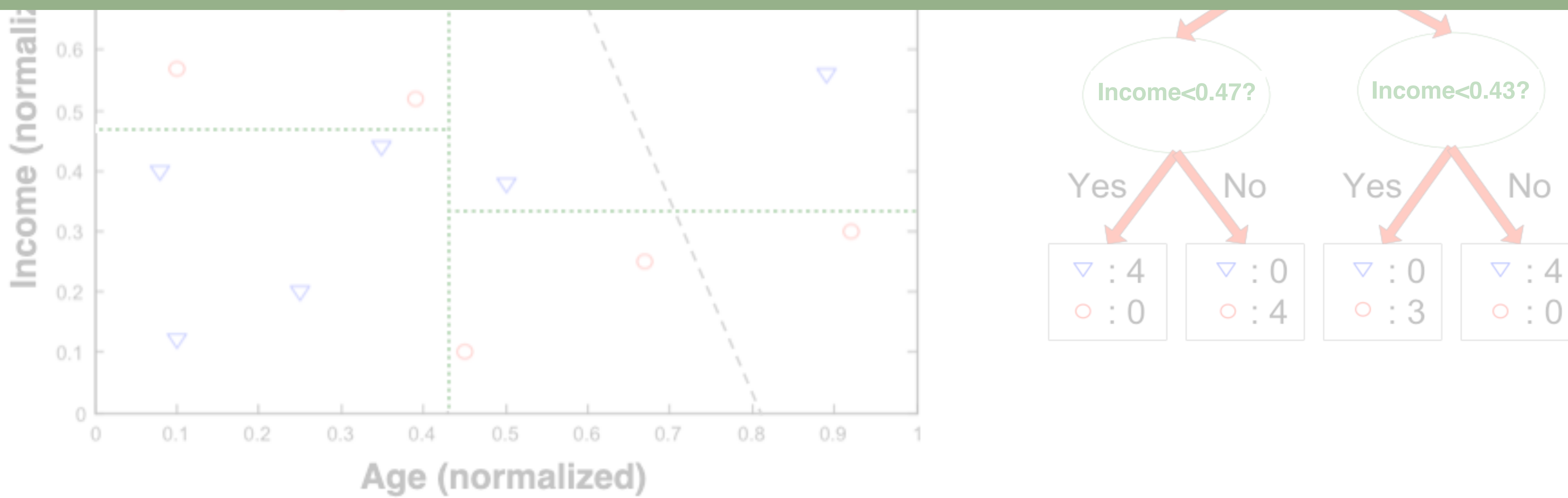
How to construct a decision tree based on training data?



Decision Trees

- General idea
  - **Repeat** "until classifier is good enough"
    - Select the "best" attribute
    - Split the instances based on the value of this attribute (new decision rule)

Divide-and-Conquer Strategy



Decision Trees

Divide-and-Conquer over data instances

id	Student	Age	Credit_Score	Will_Buy_Computer
$\mathbf{x}_1$	Yes	Young	Regular	Yes
$\mathbf{x}_2$	Yes	Middle Age	Excellent	Yes
$\mathbf{x}_3$	No	Young	Excellent	No
$\mathbf{x}_4$	No	Older Adult	Regular	No
$\mathbf{x}_5$	Yes	Older Adult	Excellent	Yes

Decision Trees

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Age?

Young

id	Student	Age	Credit_Score	Will_Buy_Computer
$\mathbf{x}_1$	Yes	Young	Regular	Yes
$\mathbf{x}_3$	No	Young	Excellent	Yes

Middle Age

id	Student	Age	Credit_Score	Will_Buy_Computer
$\mathbf{x}_2$	Yes	Middle Age	Excellent	Yes

Older Adult

id	Student	Age	Credit_Score	Will_Buy_Computer
$\mathbf{x}_4$	No	Older Adult	Regular	No
$\mathbf{x}_5$	Yes	Older Adult	Excellent	Yes

Learning a Decision Tree

- How to train a decision tree that correctly classifies these examples?

Name	***Age***	***Gender***	***TrafficTicket***	**Class: High-Risk Driver**
John	43	M	Yes	**High Risk**
Peter	18	M	No	**Low Risk**
Anna	35	F	No	**Low Risk**
Paula	19	F	No	**Low Risk**
Mark	90	M	Yes	**High Risk**
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Which attribute to test to determine a driver's label/class?

Learning a Decision Tree

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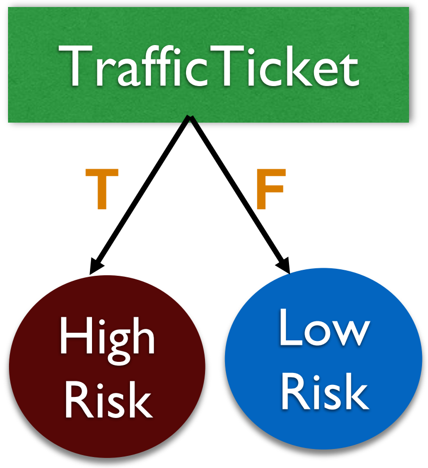

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But what if the training set is not so "well behaved"?

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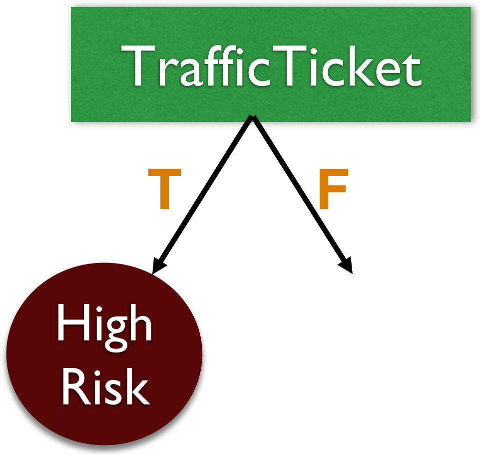

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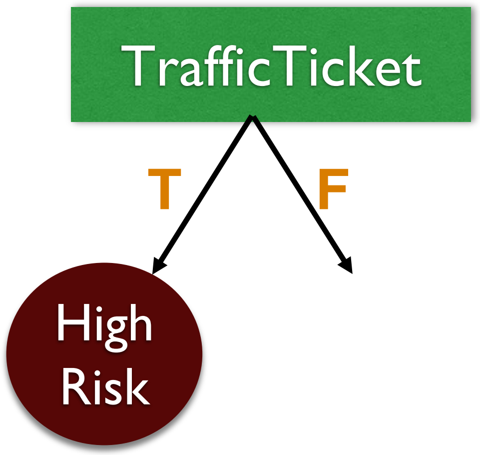

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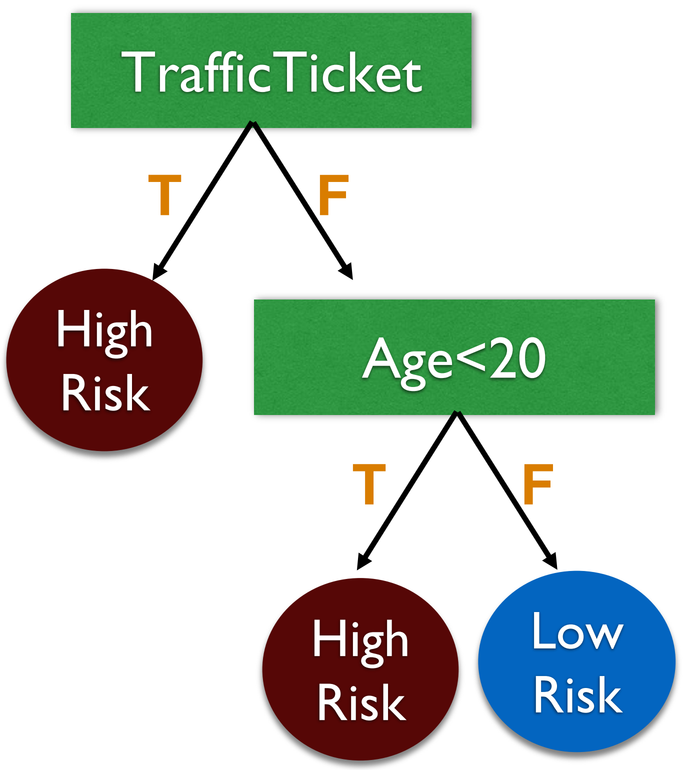

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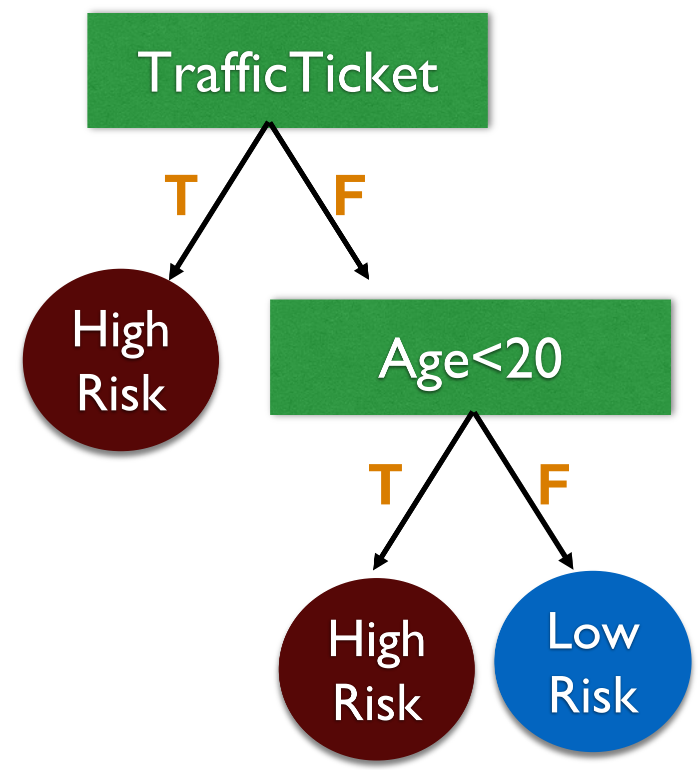

How to determine which attributes to test at each step along the tree?

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Learning a Decision Tree

- General procedure to create a decision tree
  1. Select an attribute to add to the tree (starting from the root) → new node
  2. Add, to this node, one branch for each possible value of the selected attribute
  3. Partition the instances/examples — assign each instance to its corresponding branch, based on the value of that instance's attribute
  4. Repeat these steps, recursively, for each resulting partition (i.e., for each children node)

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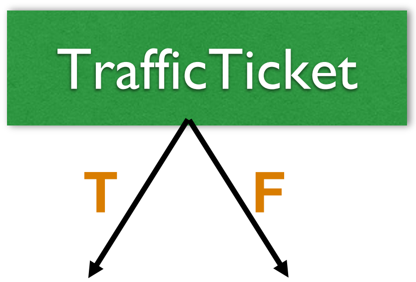
{John=**H**, Peter=**H**, Paula=**H**, Mark=**H**, Marisa=**H**, Anna=**L**, Bob=**L**}



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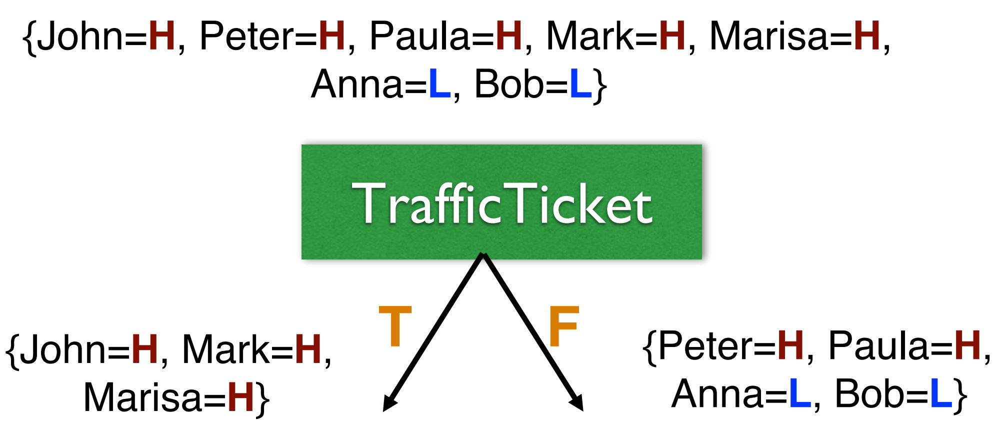

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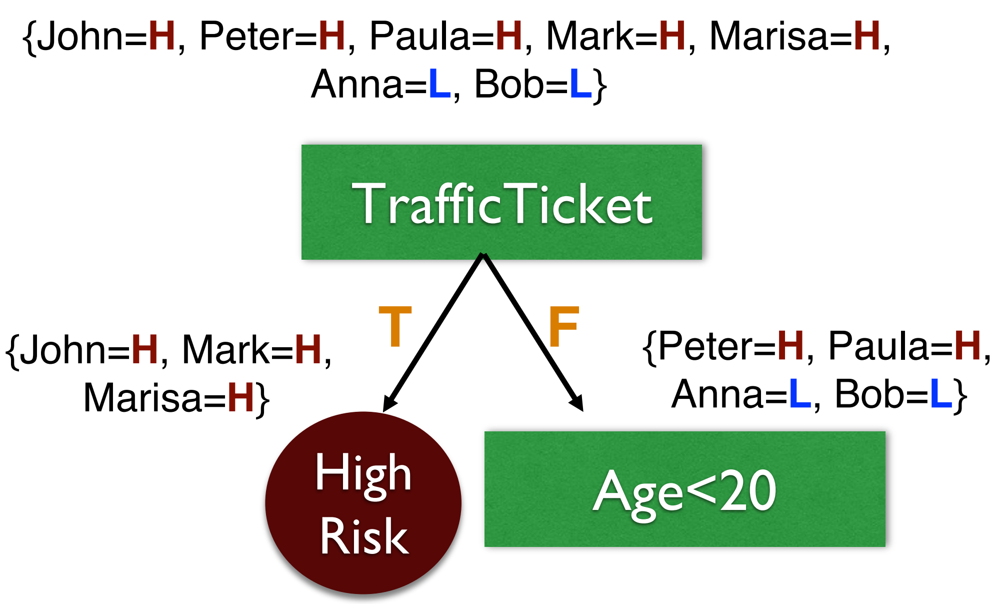

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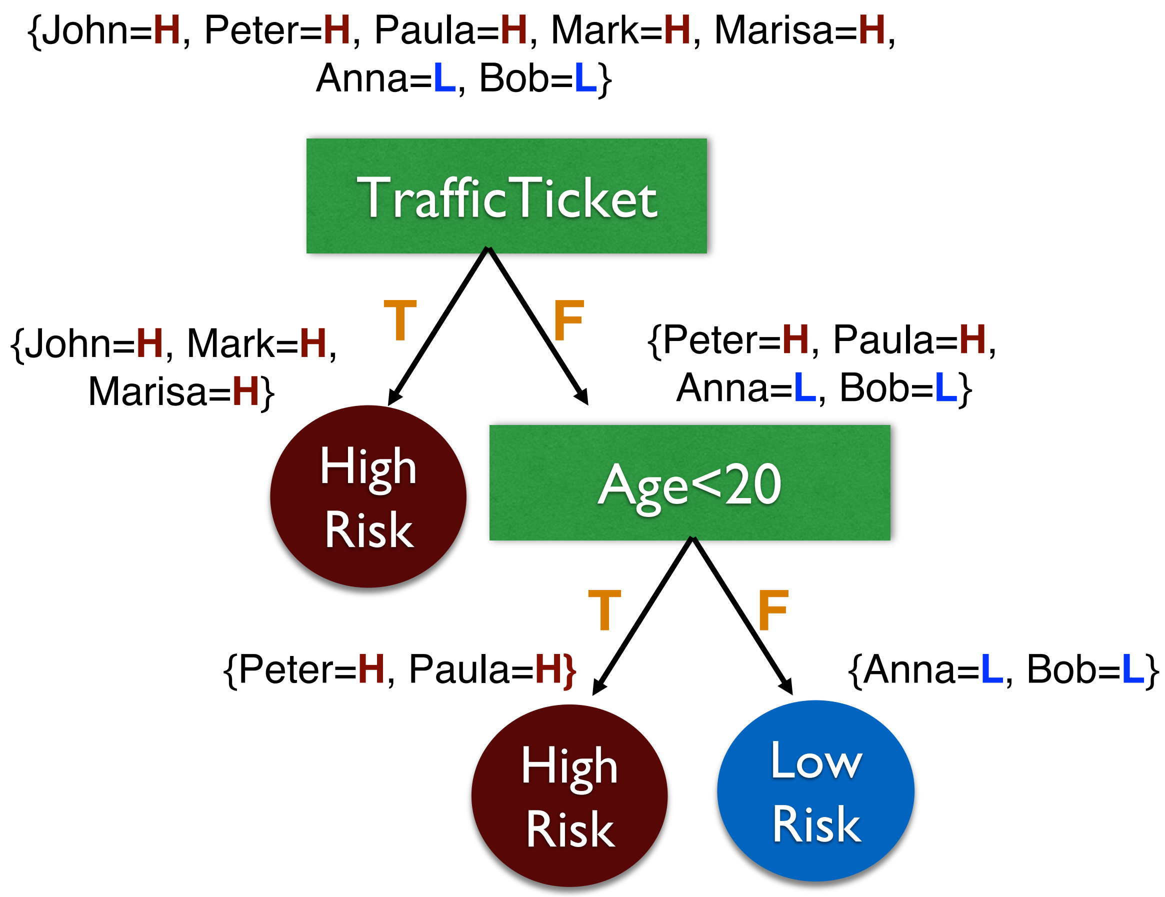

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When to stop?

a. When all instances of a given partition/node belong to the same class
   Then, create a leaf node labeled with that class

b. When there are no more attributes that can be tested
   or
   When a partition is empty (i.e., there are no instances associated with it)
   Then, create a leaf node labeled with the majority class among the instances

Learning a Decision Tree

- General procedure to create a decision tree

Select an attribute

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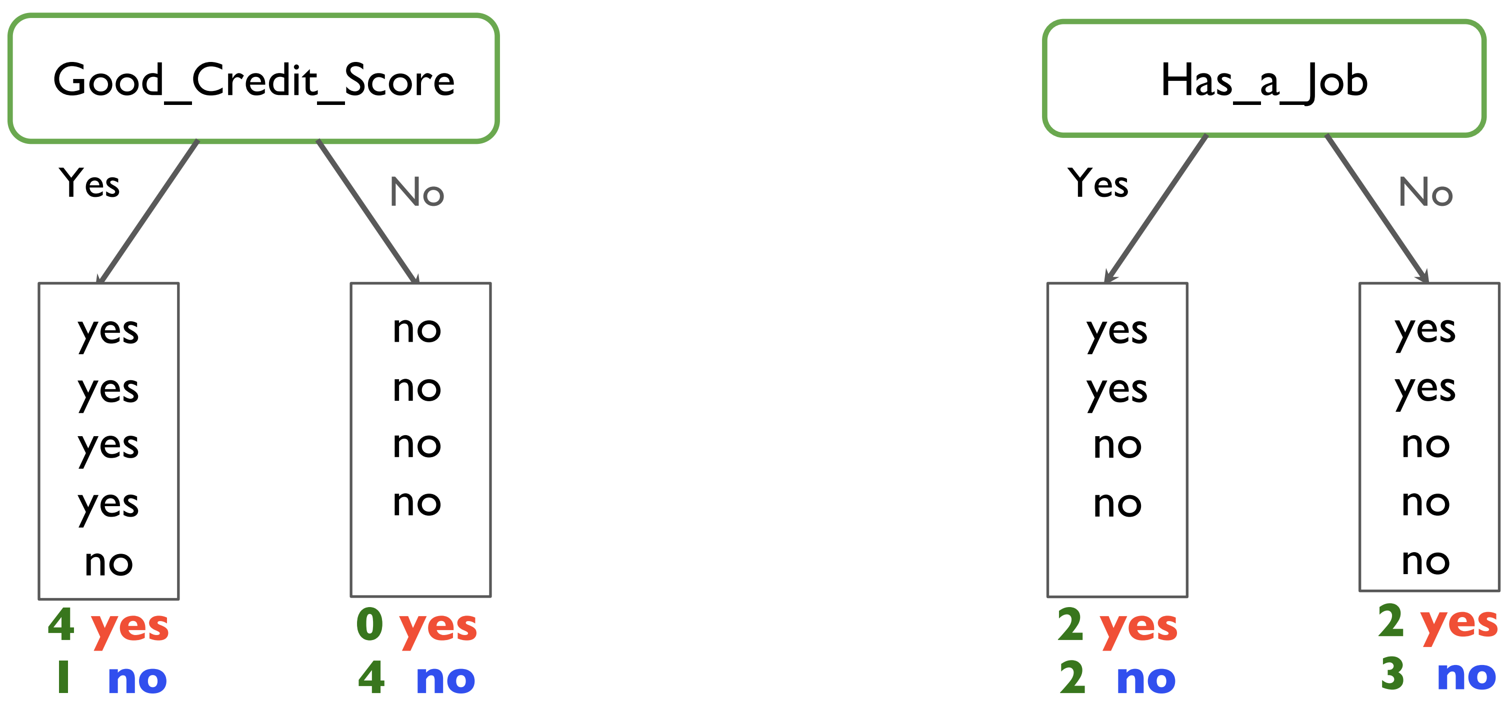
Selecting an Attribute to Test

- Criterion/heuristic for selecting which attribute to test
  - Most informative attribute
    - attribute that best splits instances according to their classes
  - Ideally, we should select an attribute such that
    - "all instances of class A go to one branch, all instances of class B to the other branch"
    - i.e., attribute that results in partitions whose instances are as homogenous as possible
      - all instances in that partition belong to the same class (in that case, add new leaf node)



"Good_Credit_Score?" seems to be more informative than "Has_a_Job"

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"Goodness of Split"

"Good_Credit_Score?" seems to be more informative than "Has_a_Job"

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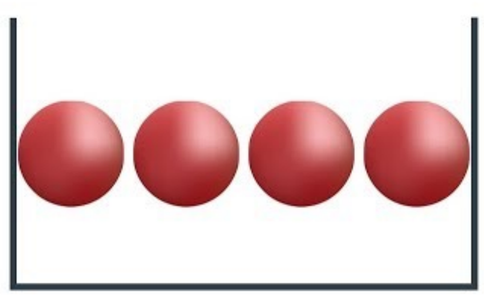
- How to quantify how homogenous a set of instances is?
  - ***Information, or entropy***
  - Information is measured in bits (or fractions of a bit)

Let's suppose we test Age, and the instances associated with Age=Young look like this


Will repay loan


Will not repay loan



Was that a useful attribute to test?

Do we have a good idea about whether the person is going to repay the loan or not?

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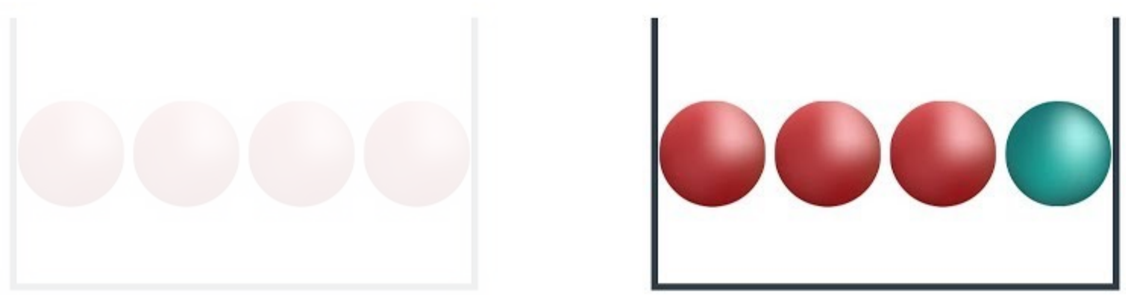

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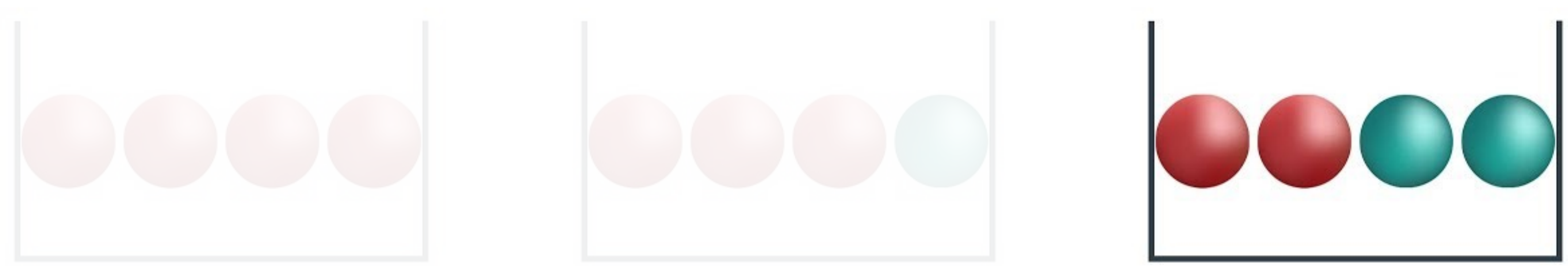
- How to quantify how homogenous a set of instances is?
  - ***Information, or entropy***
  - Information is measured in bits (or fractions of a bit)

Let's suppose we test Age, and the instances associated with Age=Young look like this

Will repay loan

Will not repay loan



Was that a useful attribute to test?

Do we have a good idea about whether the person is going to repay the loan or not?

Selecting an Attribute to Test

- Ideally, we should select an attribute such that
  - "all instances of class A go to one branch, all instances of class B to the other branch"
  - i.e., attribute that results in partitions whose instances are as homogenous as possible

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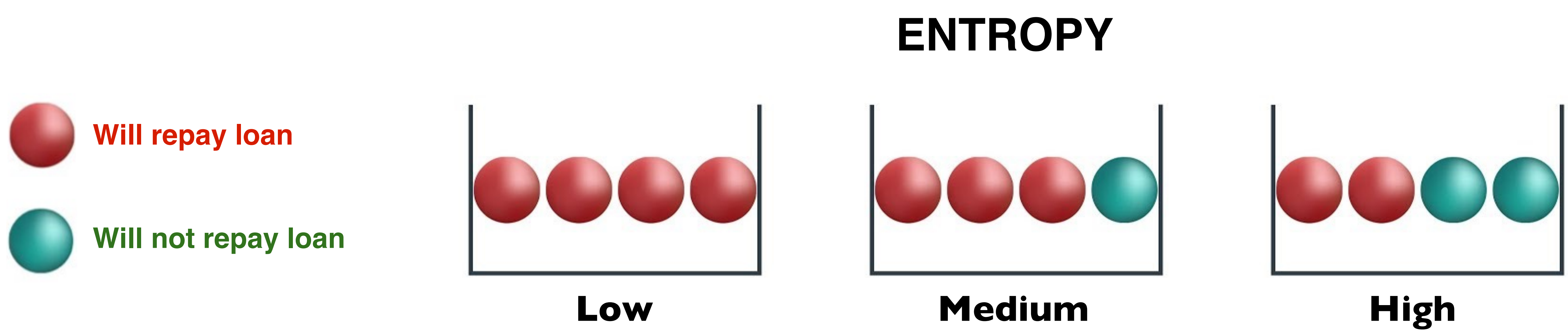

Selecting an Attribute to Test

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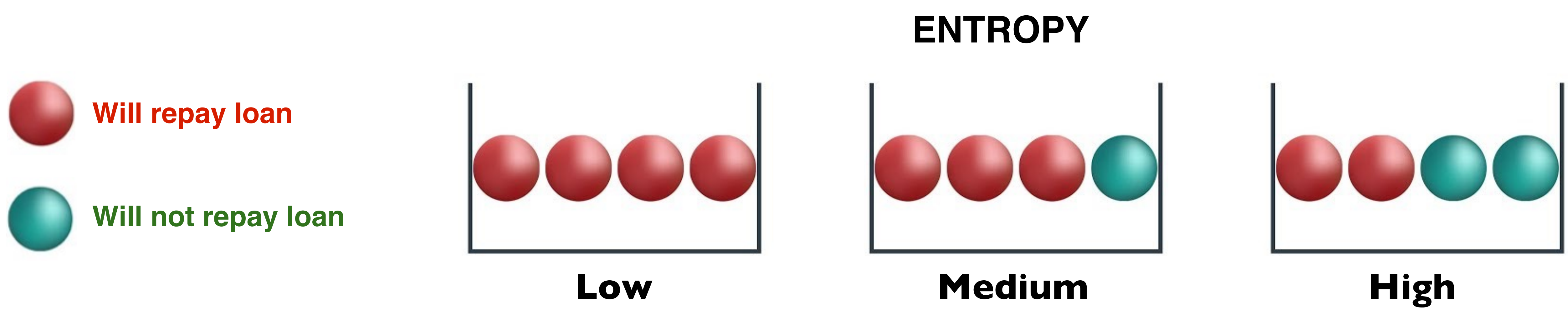

- Intuitively, quantifies how random a given quantity (e.g., class) is within a dataset
- Associated with how hard it is to predict the class based on an attribute
- Higher entropy
  → instances of a same class are all mixed up
  → testing the attribute that resulted in that partition of the data was not very useful

Selecting an Attribute to Test

- How to quantify how homogenous a set of instances is?
  - ***Information, or entropy***
  - Information is measured in bits (or fractions of a bit)

- Given a distribution of labels/classes in a partition of the data
  - how much information is required to predict the class
  - this is the *entropy* of that distribution

$$I(p_1, p_2, \ldots, p_n) = -p_1 \log_2(p_1) - p_2 \log_2(p_2) \ldots - p_n \log_2(p_n)$$

{John=**H**, Peter=**H**, Paula=**H**, Mark=**H**, Marisa=**H**,
Anna=**L**, Bob=**L**}

Selecting an Attribute to Test

- How to quantify how homogenous a set of instances is?
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Probability that class #1 (H) appears in the partition of the data

{John=**H**, Peter=**H**, Paula=**H**, Mark=**H**, Marisa=**H**, Anna=**L**, Bob=**L**}

Selecting an Attribute to Test

- How to quantify how homogenous a set of instances is?
  - ***Information, or entropy***
  - Information is measured in bits (or fractions of a bit)

- Given a distribution of labels/classes in a partition of the data
  - how much information is required to predict the class
  - this is the *entropy* of that distribution

$$I(p_1, p_2, \ldots, p_n) = -p_1 \log_2(p_1) - p_2 \log_2(p_2) \ldots - p_n \log_2(p_n)$$

Probability that class #2 (L) appears in the partition of the data

{John=**H**, Peter=**H**, Paula=**H**, Mark=**H**, Marisa=**H**, Anna=**L**, Bob=**L**}

Selecting an Attribute to Test

- How to quantify how homogenous a set of instances is?
  - ***Information, or entropy***
  - Information is measured in bits (or fractions of a bit)

- Given a distribution of labels/classes in a partition of the data
  - how much information is required to predict the class
  - this is the *entropy* of that distribution

High entropy because the class is completely undetermined (50% instances are H, 50% instances are L)

$$I(p_1, p_2, \ldots, p_n) = -p_1 \log_2(p_1) - p_2 \log_2(p_2) \ldots - p_n \log_2(p_n)$$

{John=H, Peter=H, Anna=L, Bob=L}

Pr(H) = 2/4
Pr(L) = 2/4

I(2/4, 2/4) **=** -2/4 **log₂**(2/4) -2/4 **log₂**(2/4)

= 1

Selecting an Attribute to Test

- How to quantify how homogenous a set of instances is?
  - ***Information, or entropy***
  - Information is measured in bits (or fractions of a bit)
- Given a distribution of labels/classes in a partition of the data
  - how much information is required to predict the class
  - this is the *entropy* of that distribution

$$I(p_1, p_2, \ldots, p_n) = -p_1 \log_2(p_1) - p_2 \log_2(p_2) \ldots - p_n \log_2(p_n)$$

Low entropy because the class is completely determined (100% instances are H!)

{John=**H**, Peter=**H**}

Pr(H) = 2/2
Pr(L) = 0/2

I(2/2, 0/2) = -2/2 **log2**(2/2) -0/2 **log2**(0/2)

= 0

Selecting an Attribute to Test

- How to quantify how homogenous a set of instances is?
  - ***Information, or entropy***
  - Information is measured in bits (or fractions of a bit)

- Given a distribution of labels/classes in a partition of the data
  - how much information is required to predict the cla
  - this is the *entropy* of that distribution

“Medium" entropy because the class is almost determined (almost sure it is H, but there’s still some uncertainty)

$$I(p_1, p_2, \ldots, p_n) = -p_1 \log_2(p_1) - p_2 \log_2(p_2) \ldots - p_n \log_2(p_n)$$

{John=**A**, Peter=**A**, Paula=**A**, Mark=**A**, Marisa=**A**, Anna=**B**, Bob=**B**}

I(5/7, 2/7) = -5/7 **log**$_2$(5/7) -2/7 **log**$_2$(2/7) = 0.8631

Pr(A) = 5/7
Pr(B) = 2/7

Selecting an Attribute to Test

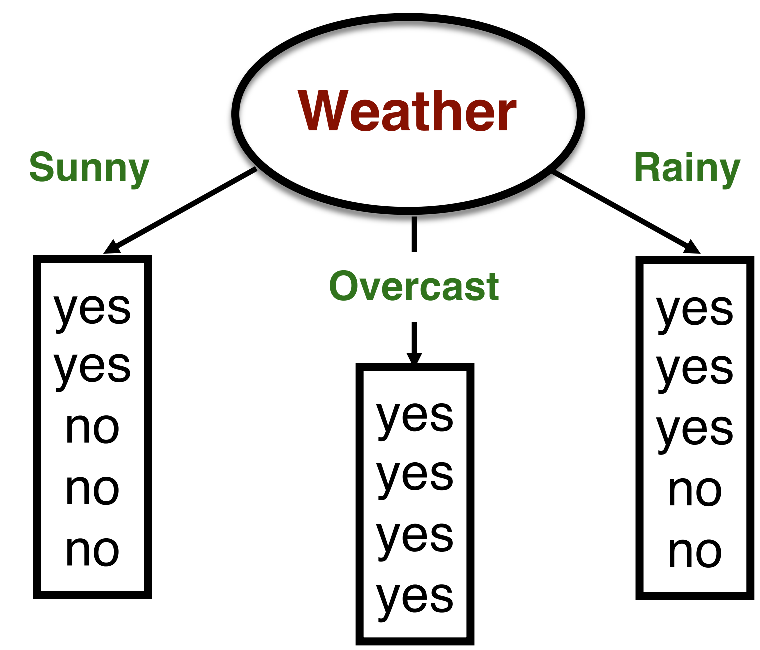
- Decision tree to predict whether a person will play tennis

Weather	Temperature	Humidity	Windy	PlayTennis
Sunny	Hot	High	False	No
Sunny	Hot	High	True	No
Overcast	Hot	High	False	Yes
Rainy	Mild	High	False	Yes
Rainy	Cool	Normal	False	Yes
Rainy	Cool	Normal	True	No
Overcast	Cool	Normal	True	Yes
Sunny	Mild	High	False	No
Sunny	Cool	Normal	False	Yes
Rainy	Mild	Normal	False	Yes
Sunny	Mild	Normal	True	Yes
Overcast	Mild	High	True	Yes
Overcast	Hot	Normal	False	Yes
Rainy	Mild	High	True	No

- Which attribute to test first?
- Let's consider testing Weather

Original dataset: 9 instances "Yes"
5 instances "No"

yes yes yes yes yes yes yes yes yes
no no no no no



Selecting an Attribute to Test

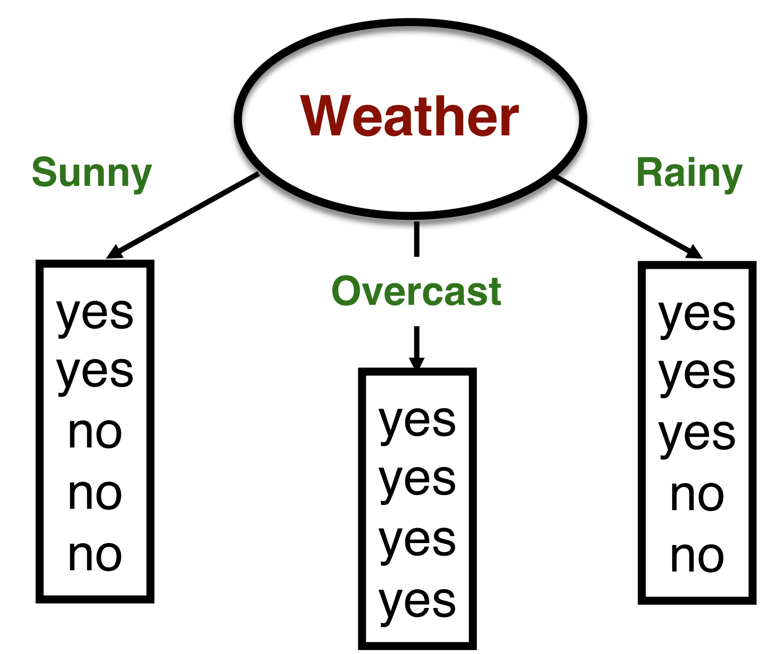
- Decision tree to predict whether a person will play tennis

- **Entropy of the original dataset:**
  - I(9/14, 5/14) = -9/14 log2(9/14) -5/14 log2(5/14)
    = **0.940 bits**
- **Entropy of partitions resulting from testing Weather:**
  - **Weather=Sunny**
    - I(2/5, 3/5) = -2/5 log2(2/5) -3/5 log2(3/5)
      = 0.971 bits
  - **Weather=Overcast**
    - I(4/4, 0/4) = -4/4 log2(4/4) -0/4 log2(0/4)
      = 0 bits
  - **Weather=Rainy**
    - I(3/5, 2/5) = -3/5 log2(3/5) -2/5 log2(2/5)
      = 0.971 bits
- **Average entropy of the resulting partitions**
  - (5/14)x0.971 + (4/14)x0 + (5/14)x0.971 = **0.693 bits**

- Which attribute to test first?
- Let's consider testing Weather

Original dataset: 9 instances "Yes"
5 instances "No"

yes yes yes yes yes yes yes yes yes no no no no no



Selecting an Attribute to Test

- Decision tree to predict whether a person will play tennis

- **Entropy of the original dataset:**
  - **I**$(9/14, 5/14) = -9/14 \log_2(9/14) - 5/14 \log_2(5/14)$
    $=$ **0.940 bits**
- **Entropy of partitions resulting from testing Weather:**
  - **Weather=Sunny**
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  - **Weather=Overcast**
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      $= 0$ bits
  - **Weather=Rainy**
    - **I**$(3/5, 2/5) = -3/5 \log_2(3/5) - 2/5 \log_2(2/5)$
      $= 0.971$ bits
- **Average entropy of the resulting partitions**
  - $(5/14)\times0.971 + (4/14)\times0 + (5/14)\times0.971 =$ **0.693 bits**

By testing the attribute **Weather**, the entropy of the classes decreased by $0.940 - 0.693 =$ **0.247 bits**

Information Gain

- quantifies how much information about the class is obtained by testing a given attribute

The algorithm will test, first, the attributes that result in higher information gain

Selecting an Attribute to Test

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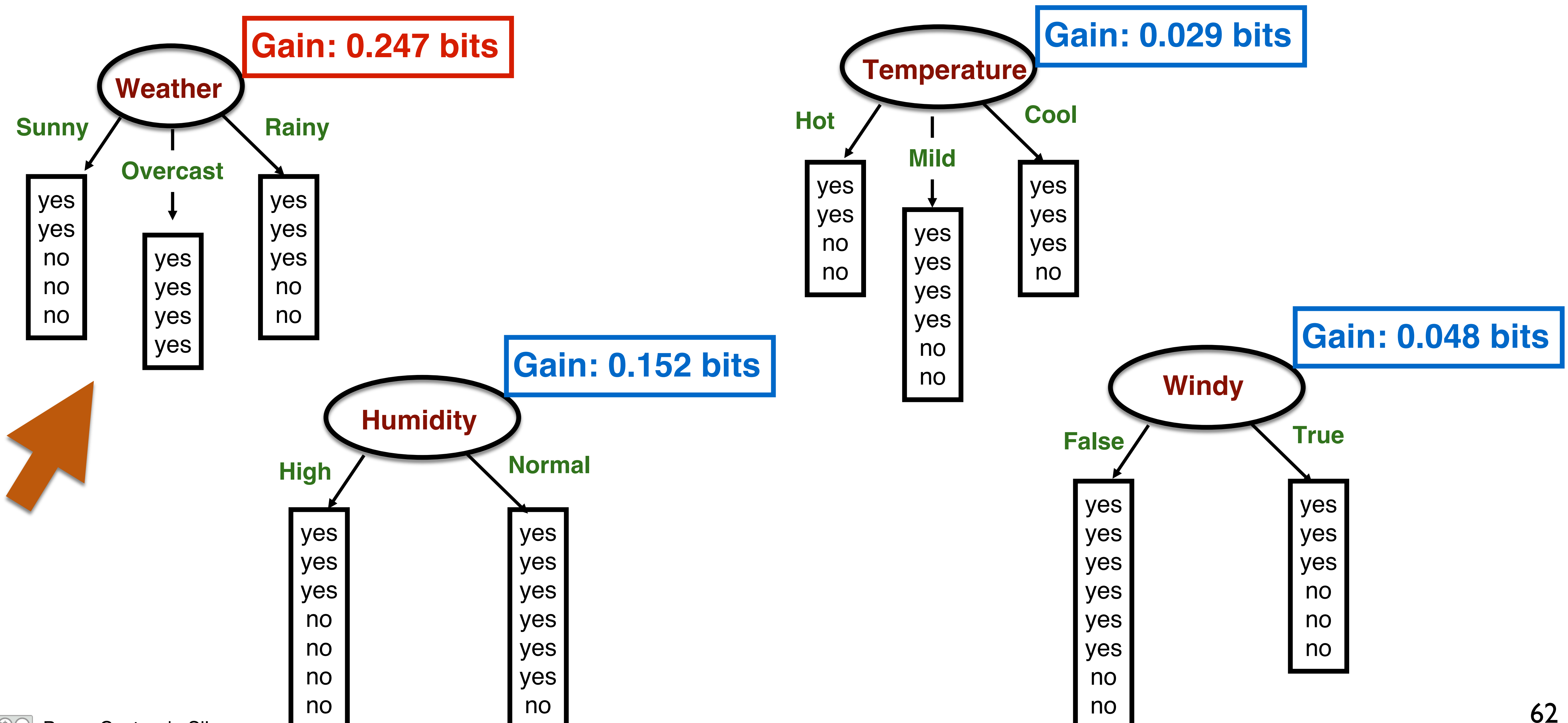
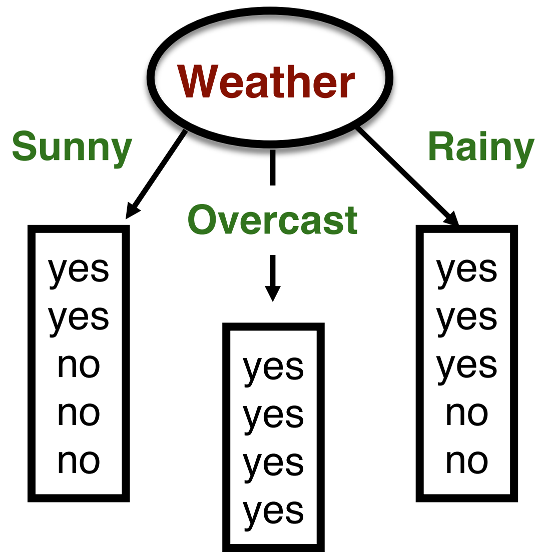
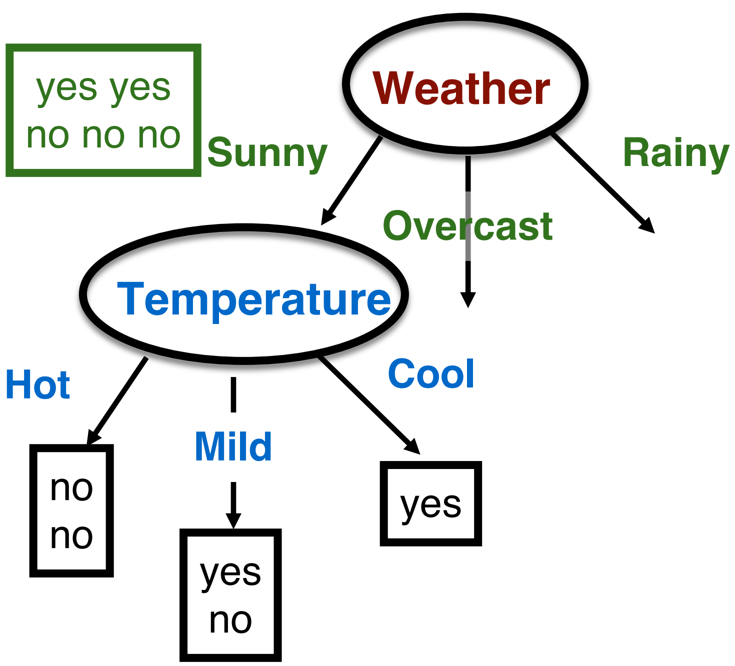


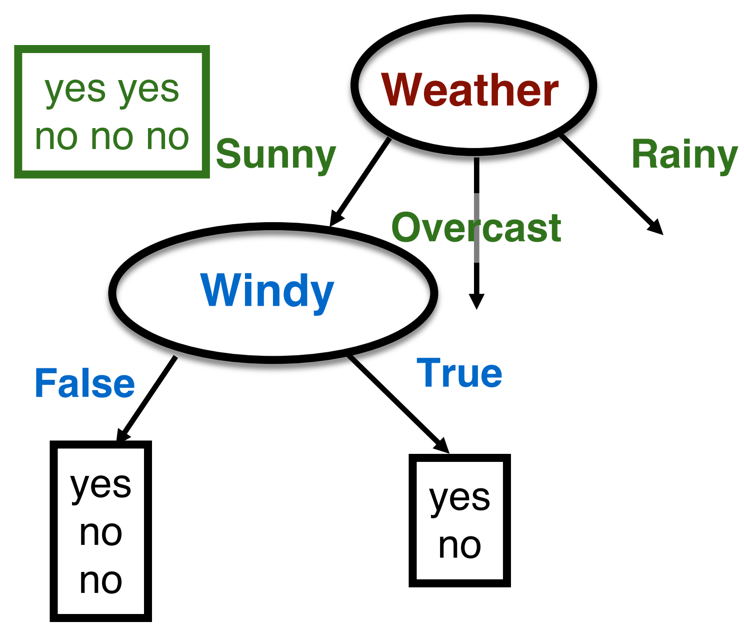
Selecting an Attribute to Test



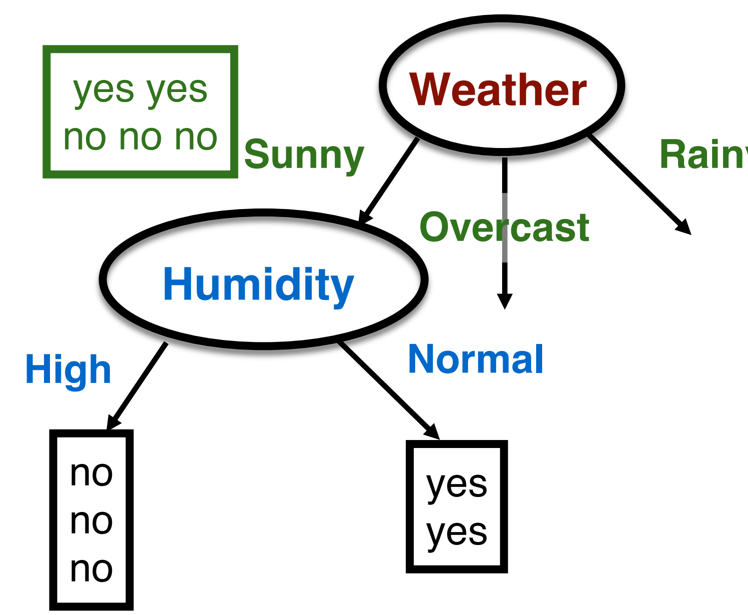
- We have decided that the 1^{st} attribute to test is Weather
- What should be tested next, on the Sunny branch?
  - i.e., should we test Temperature, Windy, or Humidity?



Gain: 0.571 bits



Gain: 0.020 bits



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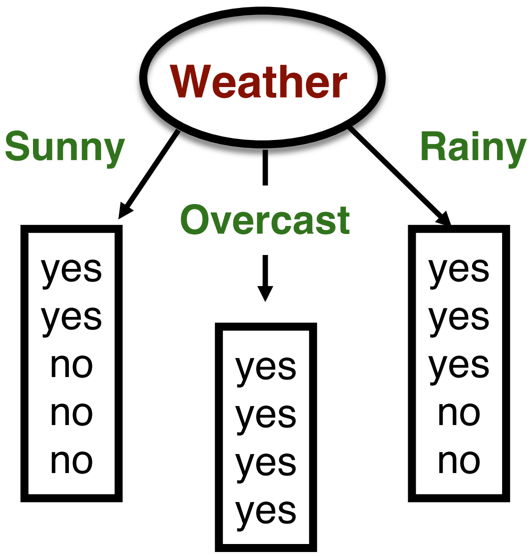
Selecting an Attribute to Test



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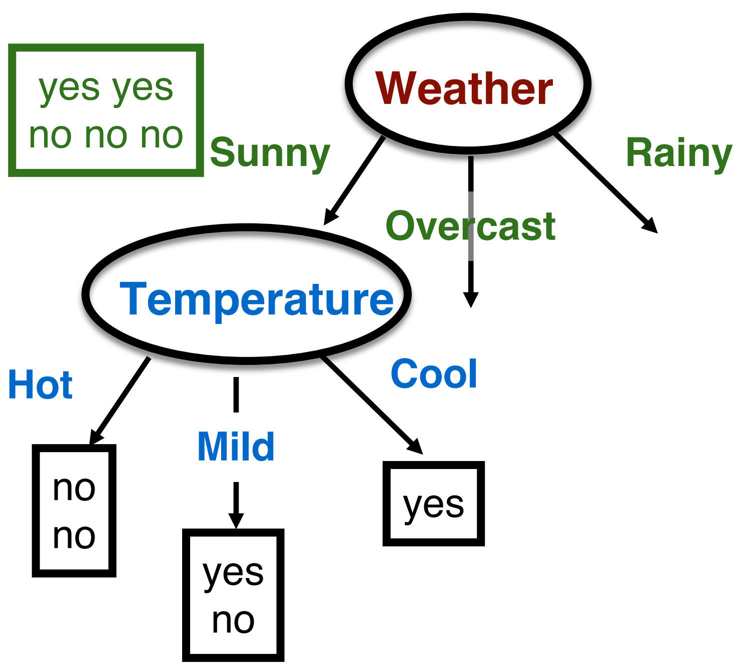

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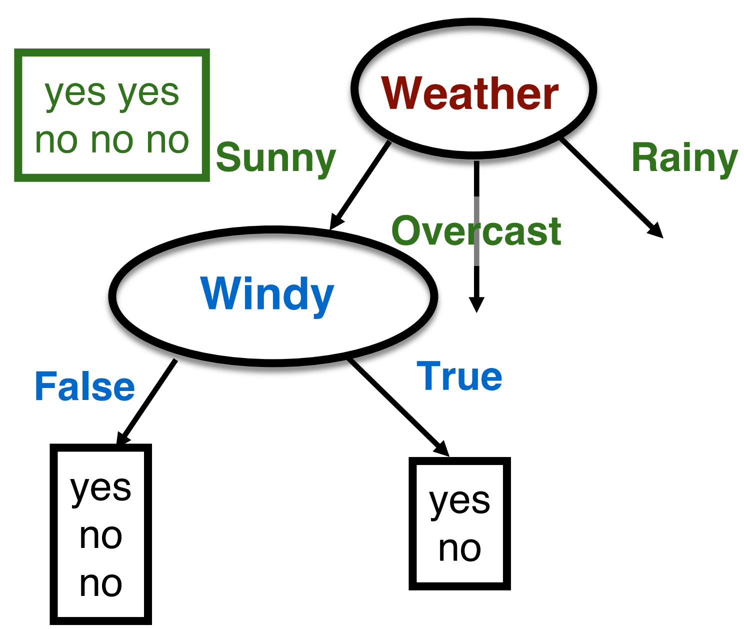

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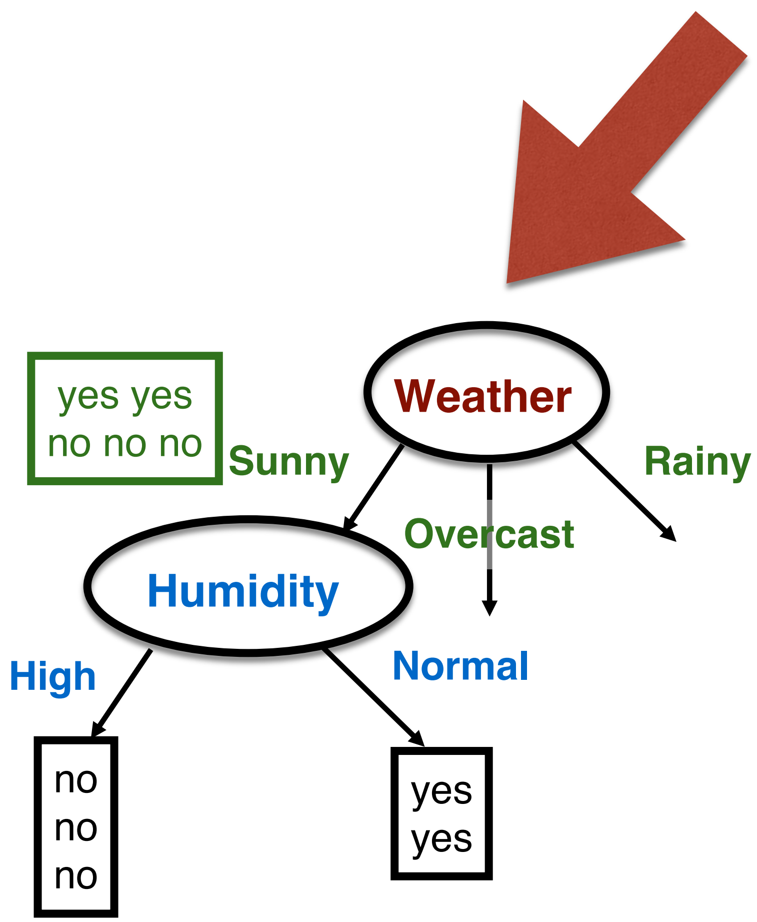

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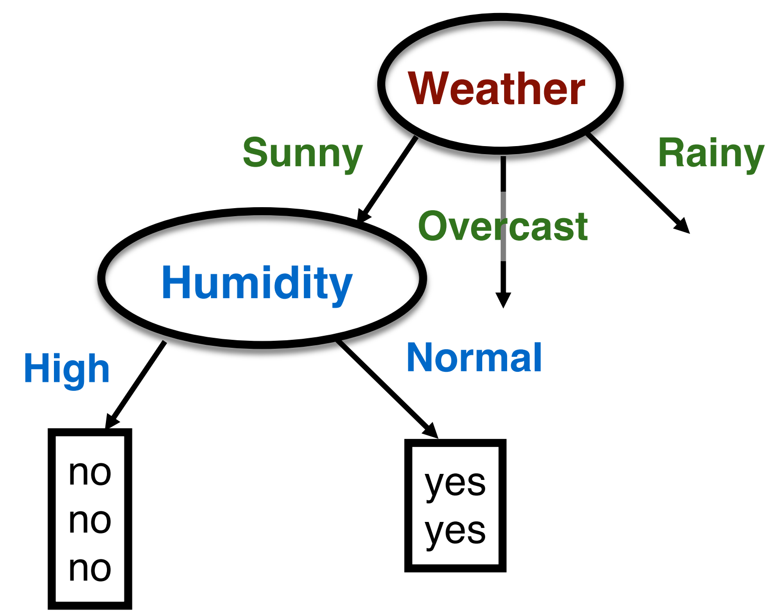

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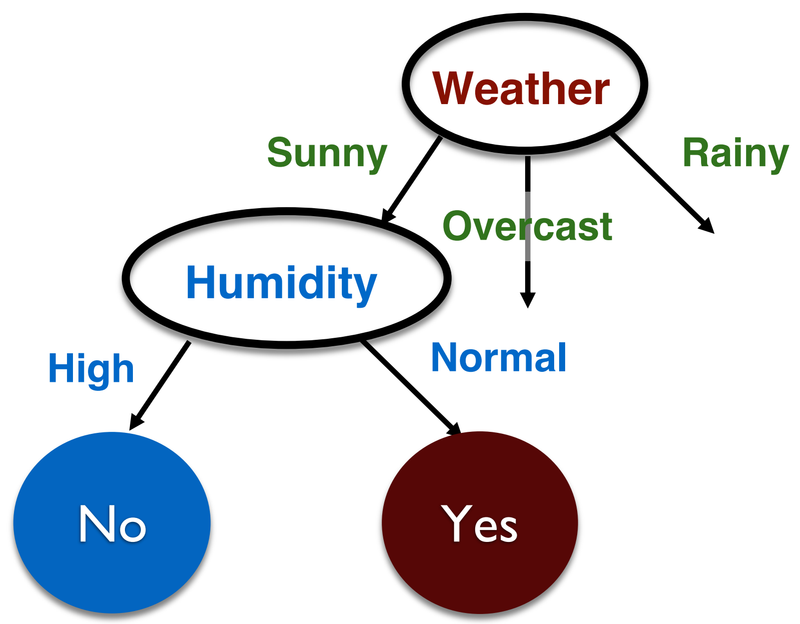

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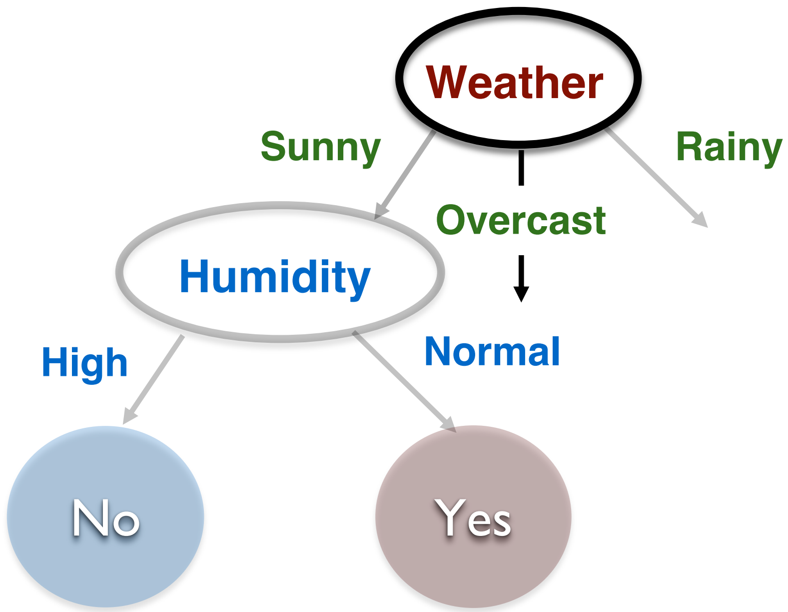

- What should be tested next, on the Overcast branch?
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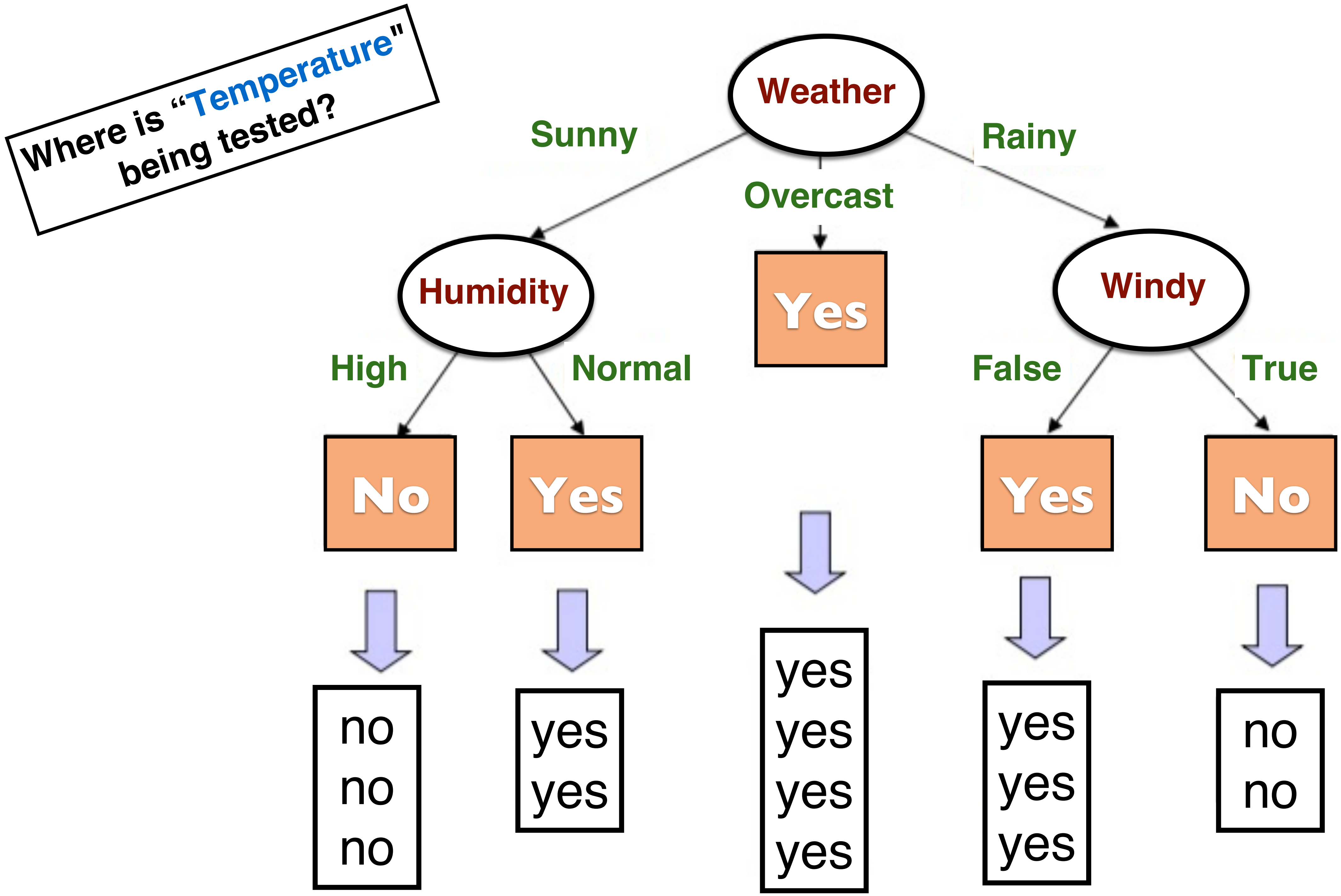
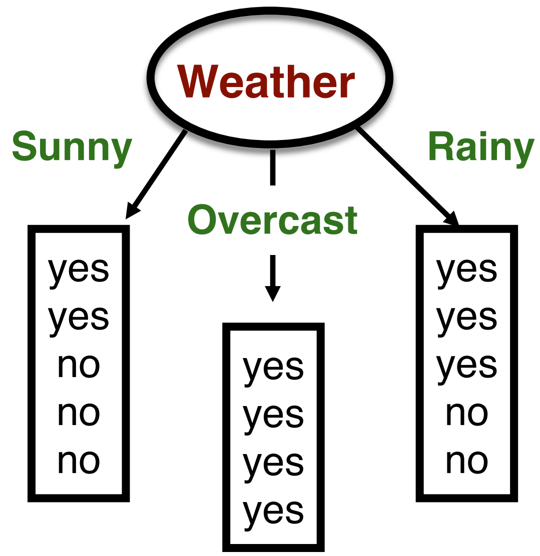
Repeat the same process, recursively...

Learned Decision Tree



Review: Selecting an Attribute to Test



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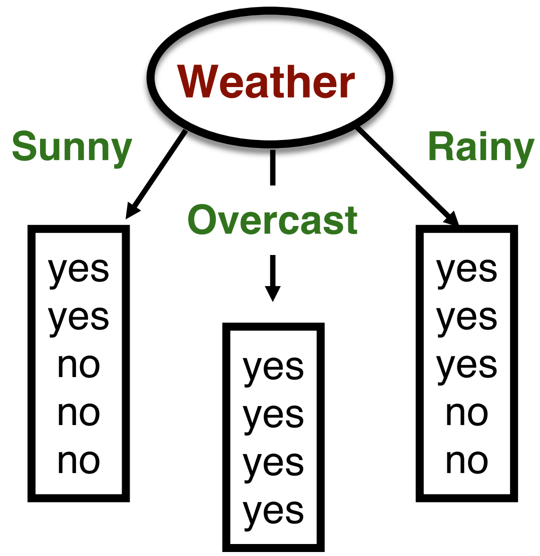

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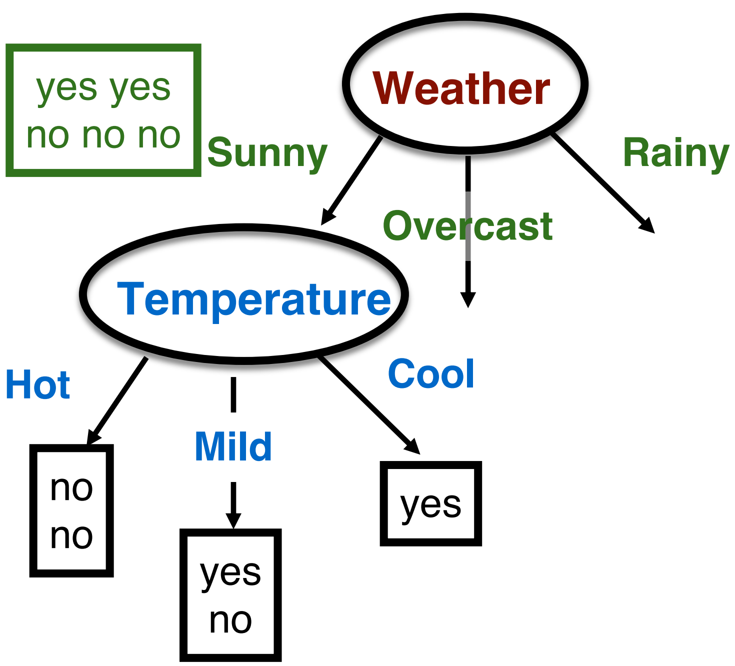

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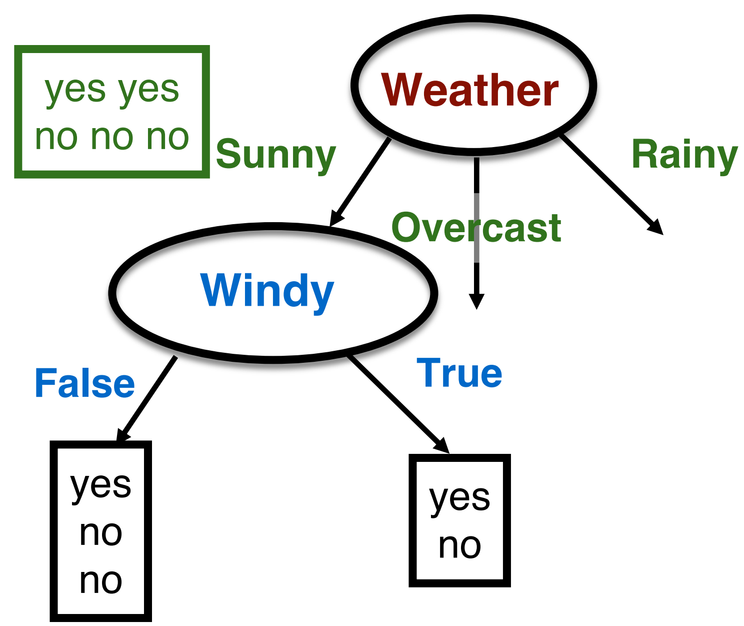

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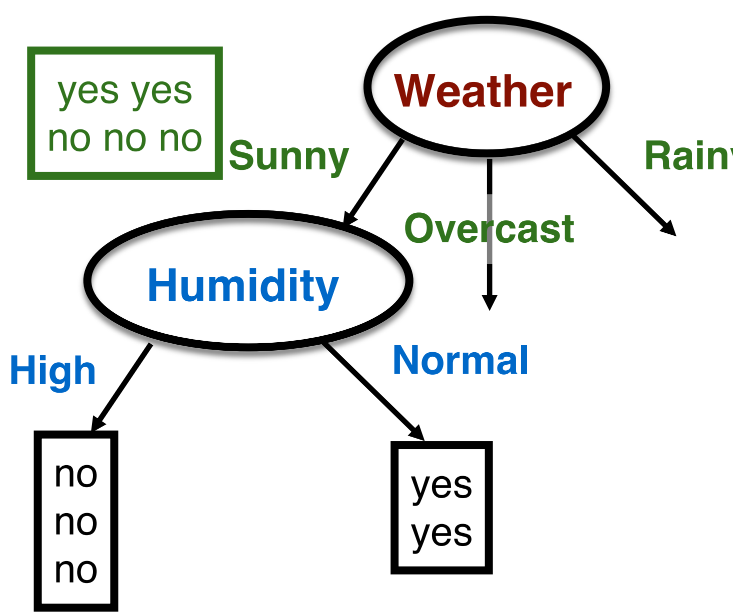

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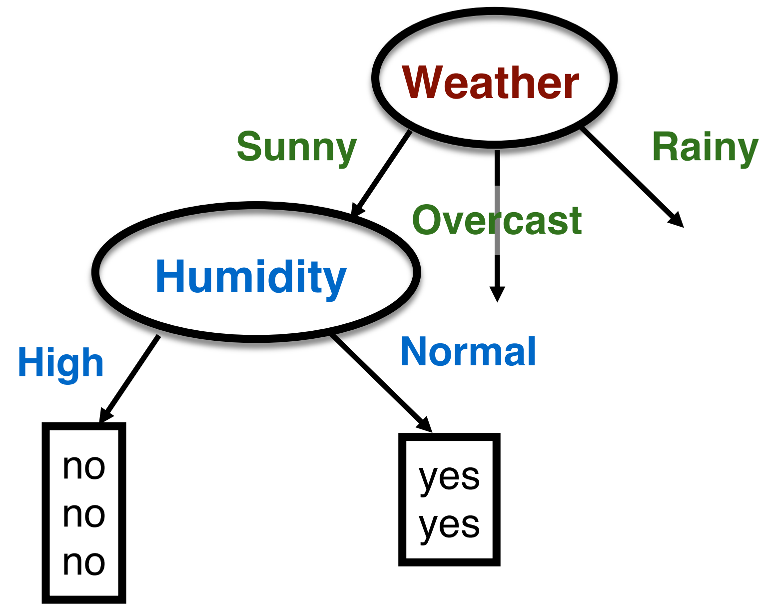

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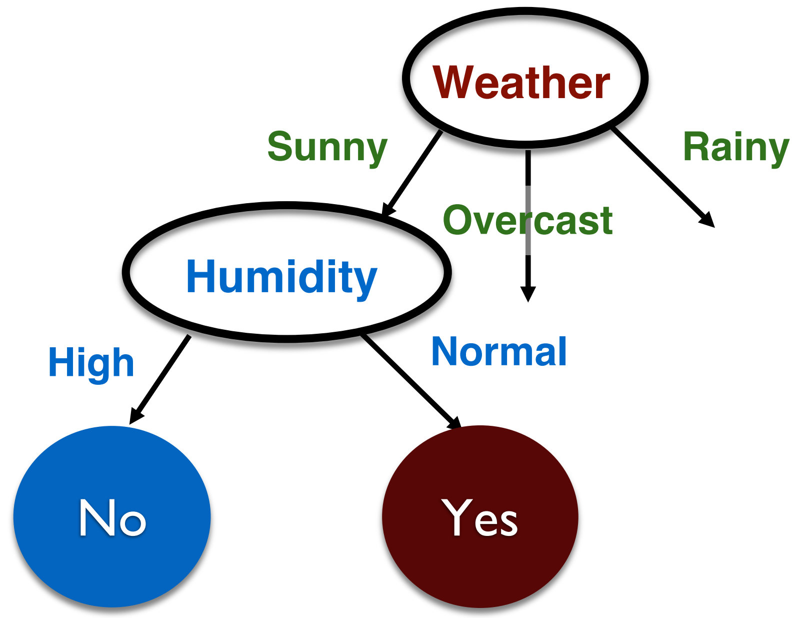

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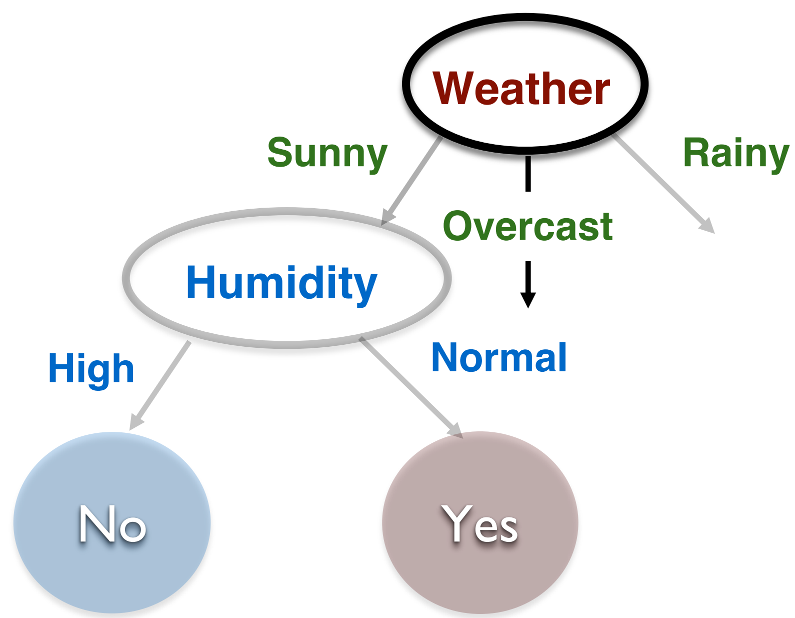

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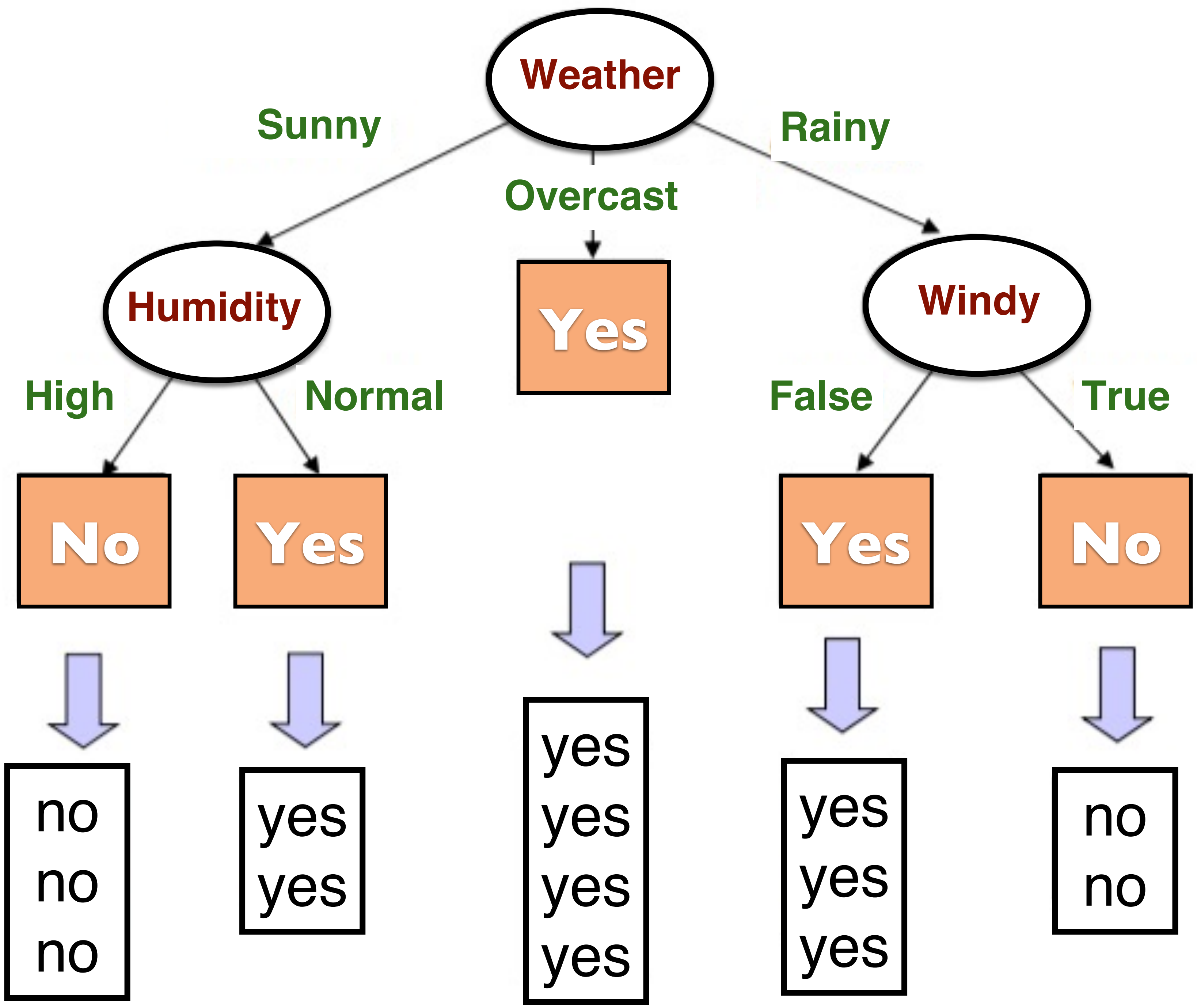

Criteria for Selecting an Attribute to Test

- We have discussed one possible criterion for selecting which attribute to test
  - **Information Gain**

- Many other criteria have been proposed — each with different properties
- Intuitively:
  - A split that keeps the same proportion of classes in each partition is **useless**
  - A split where the instances in each partition have the same class is **useful**!

- Main criteria for selecting which attribute to test:
  - **Information Gain** - ID3 Algorithm (Quilan, 1987)
  - **Information Gain Ratio** - C4.5 Algorithm (Quilan, 1988)
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All algorithms are based on the same underlying tree-learning strategy
Differ with respect to the criterion used to select which attribute to test at each point

All are greedy algorithms: select the best attribute to use when splitting a node, and never revisit this decision (no *backtracking*)

A Decision Tree Learning algorithm

Function: decision_tree(D, L)

Input: A dataset $D = \{(x_1, y_1), (x_2, y_2), \dots, (x_n, y_n)\}$ with n training instances
A list, L, of attributes that can still be tested

- Create a new node, N

- If all instances in D belong to the same class, y
  Define node N as a leaf node labeled with y and return it
- If there are no more attributes that can be tested (i.e., if $L = \emptyset$)
  Define node N as a leaf node labeled with the majority class in D, and return it

// stopping
// criteria

- Let A be the best attribute to split the dataset D **// Select splitting attribute according to some criterion**
- Define node N as a decision node that tests attribute A

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- Let A be the best attribute to split the dataset D // Select splitting attribute according to some criterion
- Define node N as a decision node that tests attribute A
- Remove A from the list of attributes that can still be tested: $L := L - \{A\}$

- Let V be a list with all different values of attribute A considering the instances in dataset D
- For each attribute value $v \in V$:
  - Let D_v be the partition of D containing all instances whose attribute $A = v$
  - If D_v is empty
    Let T_v be a leaf node labeled with the majority class in D
  - Else
    Let T_v be a sub-tree responsible for classifying the instances in D_v: $T_v :=$**decision_tree(** D_v, L **)**
  - Create an edge from node N to the root of T_v , where the edge is labeled with attribute value v

// creates
// sub-trees

- Return N

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Information Gain

- This is the criterion discussed earlier → results in a method known as **ID3**
- Intuitively, it selects the attribute A that maximizes the difference between:
  - The entropy of the original dataset D (before splitting it based on A)
  - The average entropy of the resulting partitions if we split dataset D based on A

Formally:

- Let p_i be the probability that the label i occurs in instances in a dataset D
- Let $I(D) = -\sum_{i=1}^{m} p_i \log_2(p_i)$ be the entropy of an arbitrary dataset D, where m is the number of classes/labels
- Assume that the attribute A can take up v values

  (that is, if we split D based on attribute A, we will end up with v partitions)
- Let $\text{Info}_A(D) = \sum_{j=1}^{v} \frac{|D_j|}{|D|} I(D_j)$ be the average entropy of the partitions resulting from splitting D based on A
- Let $\text{Gain}_A(D) = I(D) - \text{Info}_A(D)$ be the **Information Gain** resulting from splitting based on attribute A
- At each step, the algorithm splits the instances based on the attribute A with **highest Information Gain**

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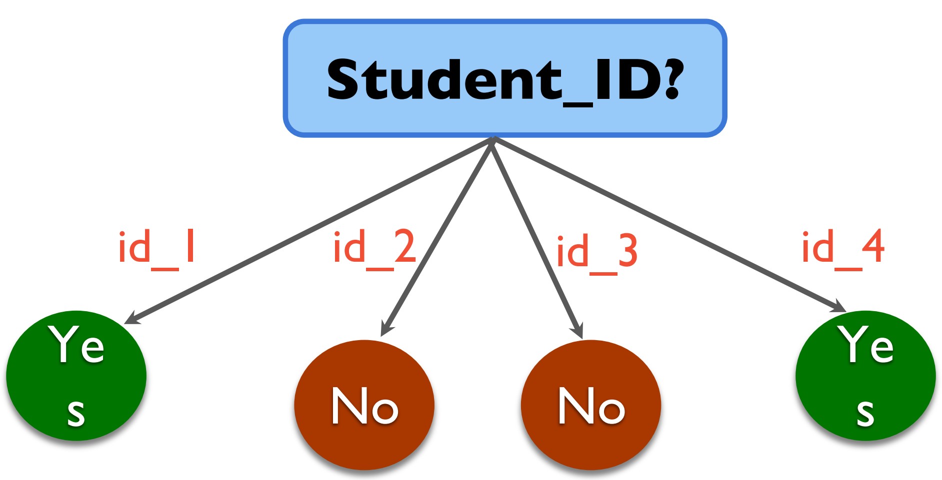
- Often results in a decision tree that is not necessarily the "simplest" one
- Intuitively, it often chooses attributes with many possible values (like Student_ID, Name, etc)

Student_ID	Student	Age	Credit_Score	Will_Buy_Computer
id_1	Yes	Young	Regular	Yes
id_2	Yes	Middle Age	Excellent	No
id_3	No	Young	Excellent	No
id_4	No	Older Adult	Regular	Yes



- Perfect split!
- With just one test, can "predict" the class perfectly
- But it is clearly overfitting ("memorizing" the dataset)

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id_1	Yes	Young	Regular	Yes
id_2	Yes	Middle Age	Excellent	No
id_3	No	Young	Excellent	No
id_4	No	Older Adult	Regular	Yes

The **Information Gain Ratio** criterion, implemented by the C4.5 algorithm, tries to mitigate this issue

Criteria for Selecting an Attribute to Test

- We have discussed <u>one</u> possible criterion for selecting which attribute to test
  - **Information Gain**

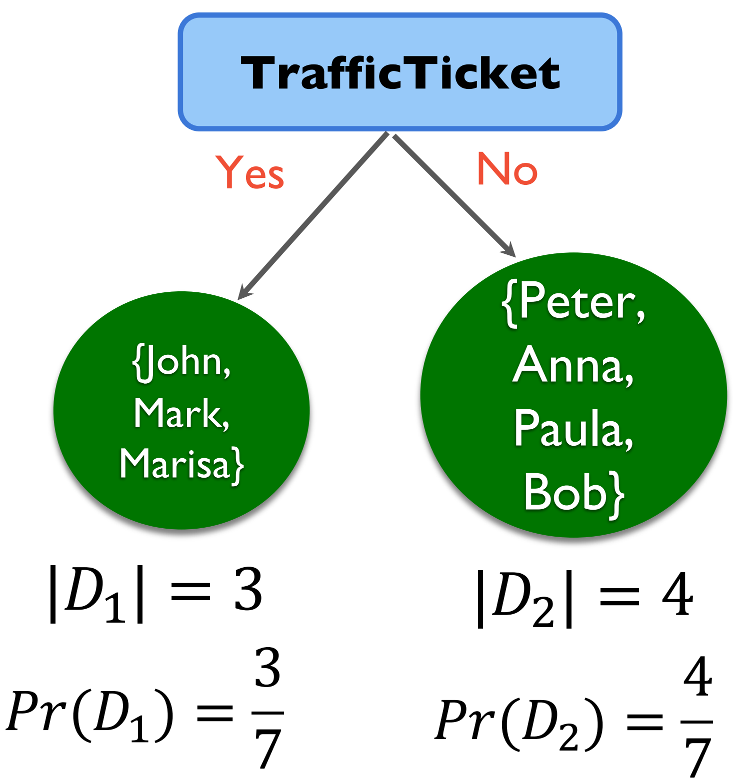
- Many other criteria have been proposed — each with different properties
- Intuitively:
  - A split that keeps the same proportion of classes in each partition is **useless**
  - A split where the instances in each partition have the same class is **useful**!

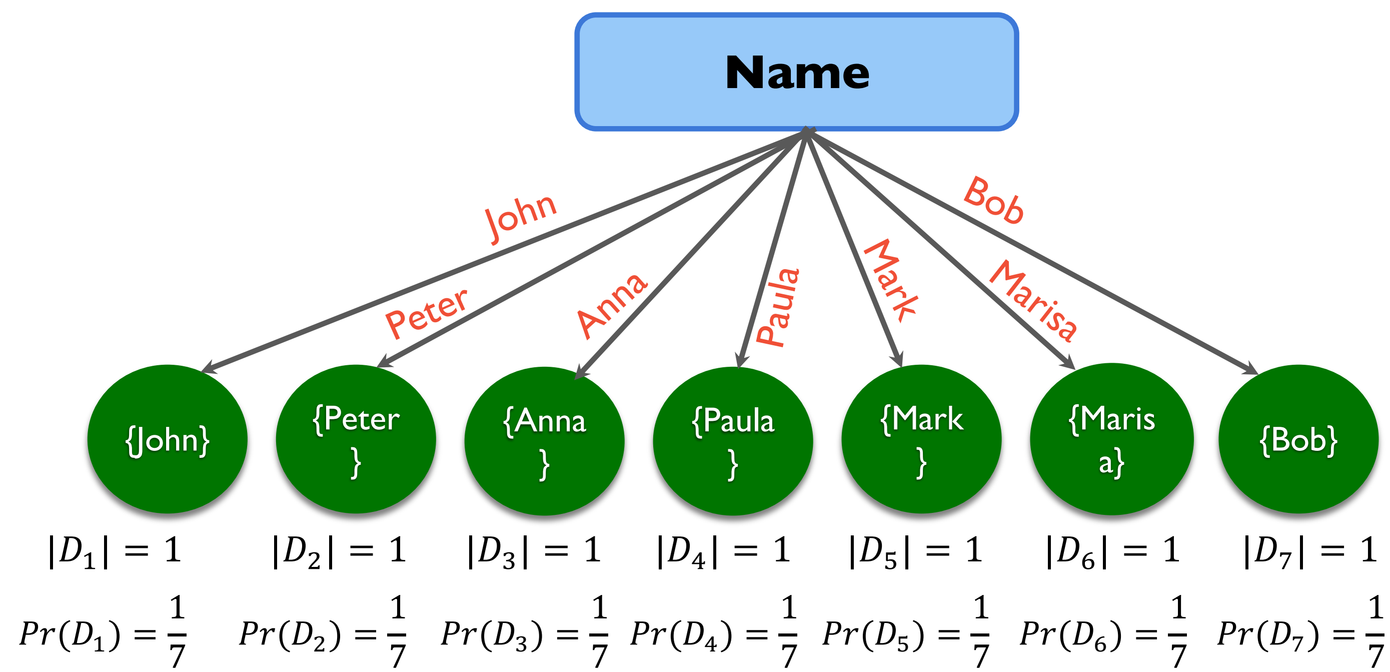
- Main criteria for selecting which attribute to test:
  - **Information Gain** - <u>ID3</u> Algorithm (Quilan, 1987)
  - **Information Gain Ratio** - <u>C4.5</u> Algorithm (Quilan, 1988)
  - **Gini Impurity** - <u>CART</u> Algorithm (Breiman, 1984)

Information Gain Ratio

Name	*Age*	*Gender*	*TrafficTicket*	Class: High-Risk Driver
John	43	M	Yes	**High Risk**
Peter	18	M	No	**Low Risk**
Anna	35	F	No	**Low Risk**
Paula	19	F	No	**Low Risk**
Mark	90	M	Yes	**High Risk**
Marisa	19	F	Yes	**Low Risk**
Bob	30	M	No	**Low Risk**

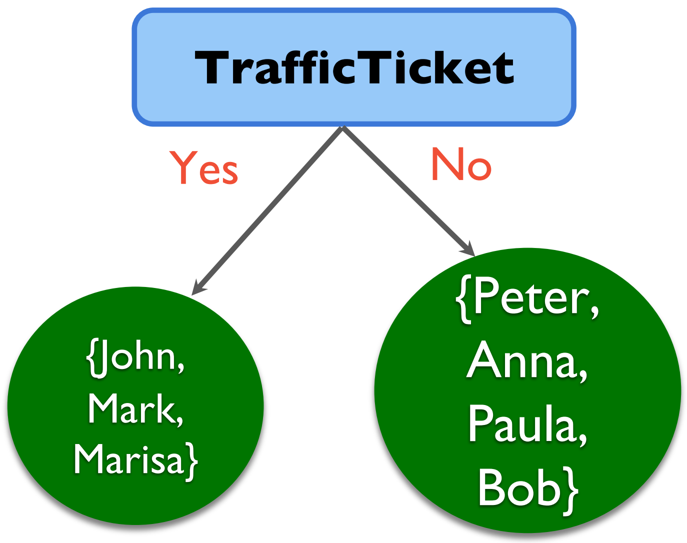
- “Adjusts" Information Gain criterion to lessen the bias towards attributes that create many branches
- Intuition:





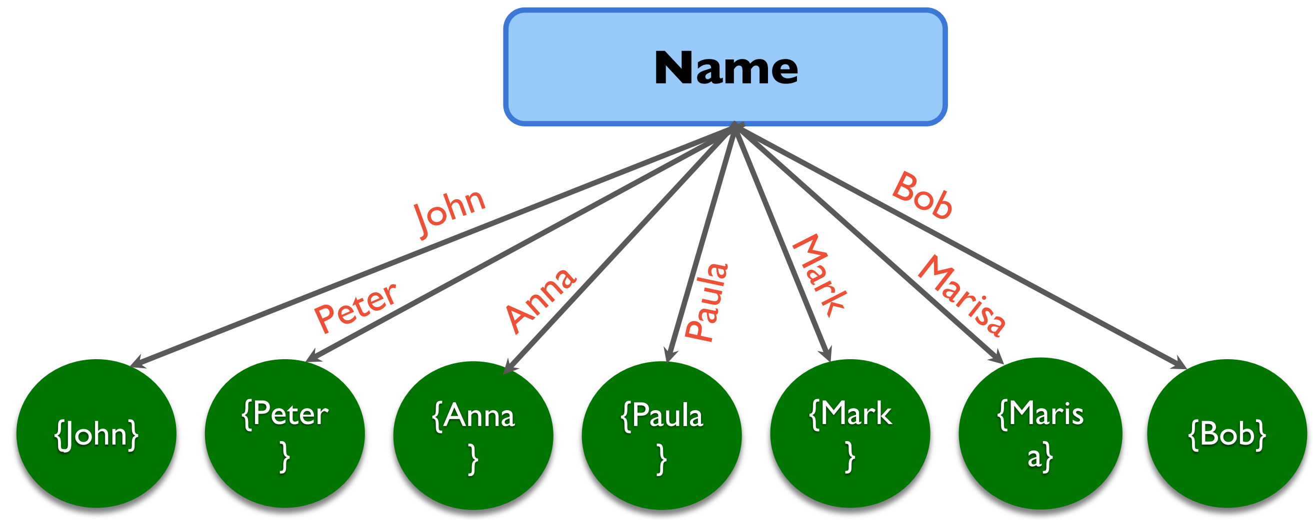
Information Gain Ratio

- "Adjusts" Information Gain criterion to lessen the bias towards attributes that create many branches
- Intuition:



$|D_1| = 3$ $\quad Pr(D_1) = \frac{3}{7}$

$|D_2| = 4$ $\quad Pr(D_2) = \frac{4}{7}$

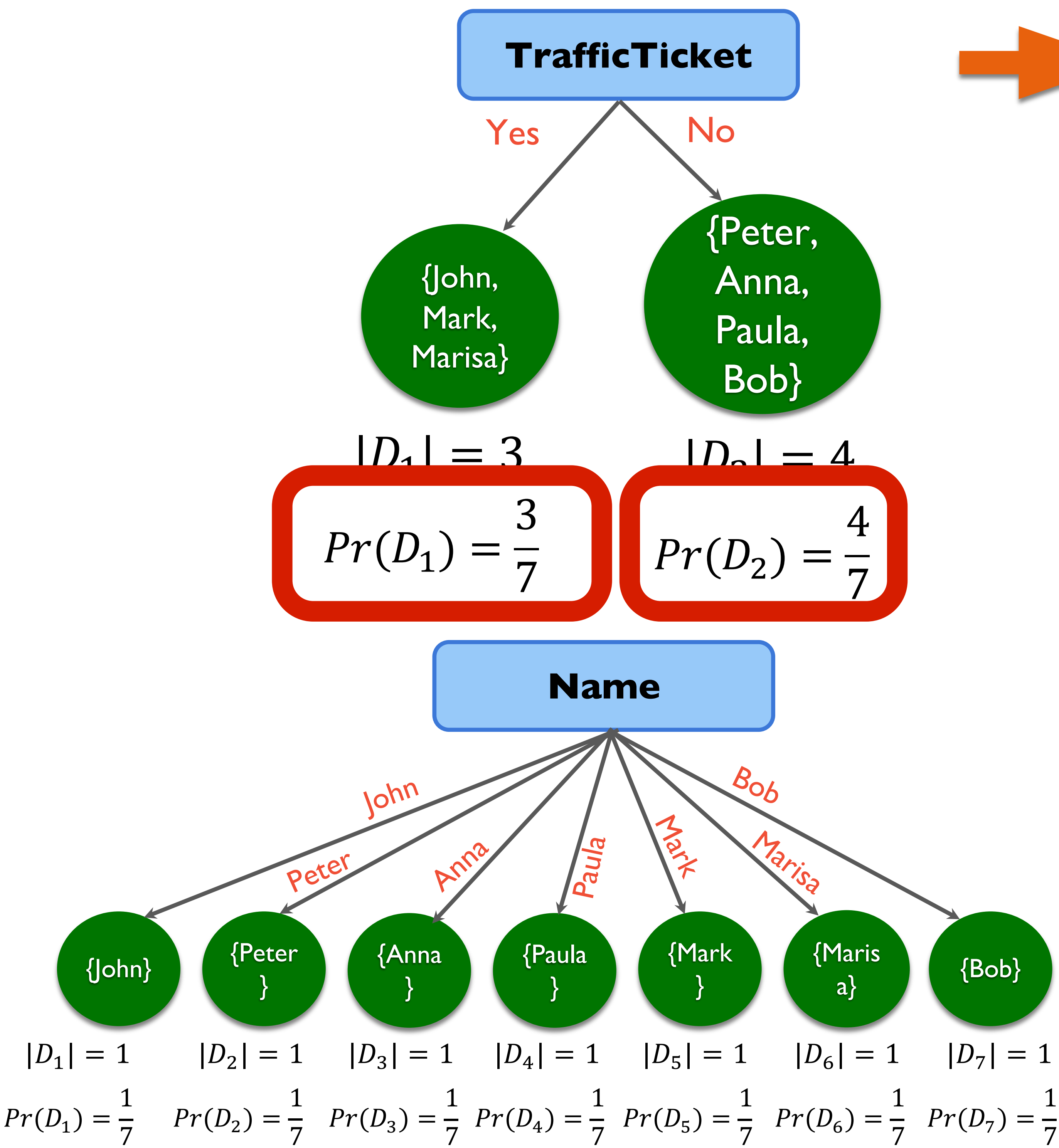


$|D_1| = 1$, $Pr(D_1) = \frac{1}{7}$; $|D_2| = 1$, $Pr(D_2) = \frac{1}{7}$; $|D_3| = 1$, $Pr(D_3) = \frac{1}{7}$; $|D_4| = 1$, $Pr(D_4) = \frac{1}{7}$; $|D_5| = 1$, $Pr(D_5) = \frac{1}{7}$; $|D_6| = 1$, $Pr(D_6) = \frac{1}{7}$; $|D_7| = 1$, $Pr(D_7) = \frac{1}{7}$

- If there are lots of branches (e.g., if we split by Name, there are as many branches as attribute values!)
  - Then these probabilities will be very similar/homogenous
- How to quantify how "homogeneous" these quantities are?
  - We've seen something like this before… Entropy!

Information Gain Ratio

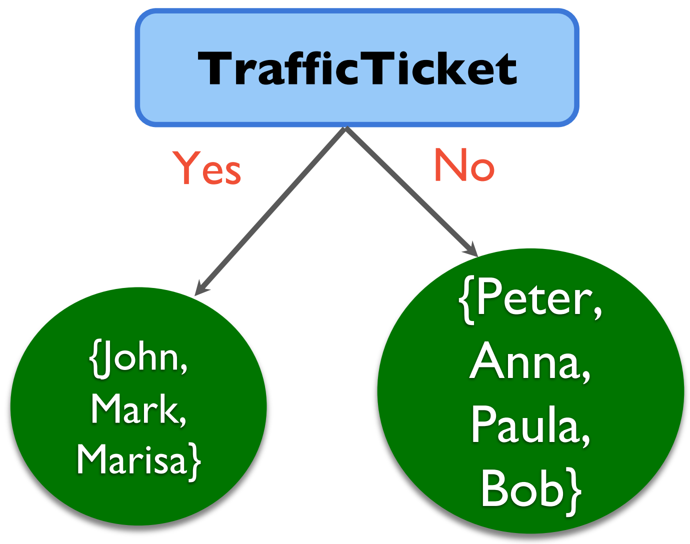
- How to quantify how "homogeneous" these quantities are?
  - We've seen something like this before… Entropy!



$\rightarrow$ $$-\frac{3}{7}\log_2\left(\frac{3}{7}\right) - \frac{4}{7}\log_2\left(\frac{4}{7}\right)$$

Information Gain Ratio

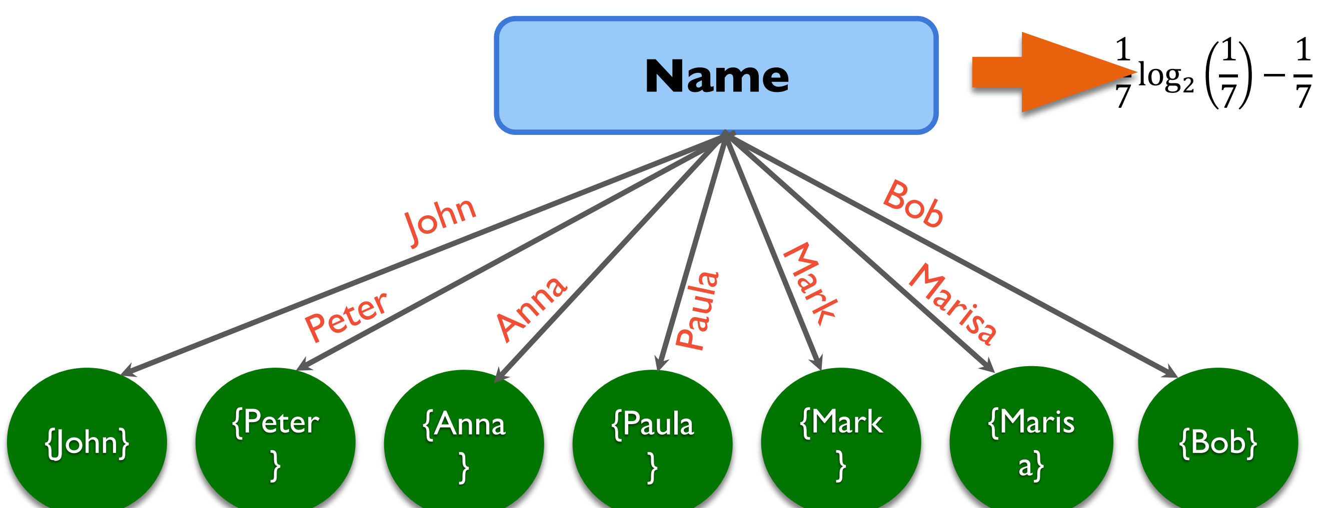
- How to quantify how “homogeneous" these quantities are?
  - We’ve seen something like this before… Entropy!



$$-\frac{3}{7}\log_2\left(\frac{3}{7}\right)-\frac{4}{7}\log_2\left(\frac{4}{7}\right)$$

$$|D_1| = 3 \qquad |D_2| = 4$$

$$Pr(D_1) = \frac{3}{7} \qquad Pr(D_2) = \frac{4}{7}$$



$$\frac{1}{7}\log_2\left(\frac{1}{7}\right)-\frac{1}{7}\log_2\left(\frac{1}{7}\right)-\frac{1}{7}\log_2\left(\frac{1}{7}\right)-\frac{1}{7}\log_2\left(\frac{1}{7}\right)-\frac{1}{7}\log_2\left(\frac{1}{7}\right)-\frac{1}{7}\log_2\left(\frac{1}{7}\right)-\frac{1}{7}\log_2\left(\frac{1}{7}\right)$$

$|D_1| = 1$ $|D_2| = 1$ $|D_3| = 1$ $|D_4| = 1$ $|D_5| = 1$ $|D_6| = 1$ $|D_7| = 1$

$\boxed{Pr(D_1) = \frac{1}{7}}$ $\boxed{Pr(D_2) = \frac{1}{7}}$ $\boxed{Pr(D_3) = \frac{1}{7}}$ $\boxed{Pr(D_4) = \frac{1}{7}}$ $\boxed{Pr(D_5) = \frac{1}{7}}$ $\boxed{Pr(D_6) = \frac{1}{7}}$ $\boxed{Pr(D_7) = \frac{1}{7}}$

Information Gain Ratio

- How to quantify how "homogeneous" these quantities are?
  - We've seen something like this before… Entropy! → which is called, in this context, **Split_Info**

$$-\frac{3}{7}\log_2\left(\frac{3}{7}\right)-\frac{4}{7}\log_2\left(\frac{4}{7}\right)$$

$|D_1| = 3$ $\quad$ $|D_2| = 4$

$Pr(D_1) = \frac{3}{7}$ $\quad$ $Pr(D_2) = \frac{4}{7}$

$$-\frac{1}{7}\log_2\left(\frac{1}{7}\right)-\frac{1}{7}\log_2\left(\frac{1}{7}\right)-\frac{1}{7}\log_2\left(\frac{1}{7}\right)-\frac{1}{7}\log_2\left(\frac{1}{7}\right)-\frac{1}{7}\log_2\left(\frac{1}{7}\right)-\frac{1}{7}\log_2\left(\frac{1}{7}\right)-\frac{1}{7}\log_2\left(\frac{1}{7}\right)$$

$|D_1| = 1$ $\quad$ $|D_2| = 1$ $\quad$ $|D_3| = 1$ $\quad$ $|D_4| = 1$ $\quad$ $|D_5| = 1$ $\quad$ $|D_6| = 1$ $\quad$ $|D_7| = 1$

$Pr(D_1) = \frac{1}{7}$ $\quad$ $Pr(D_2) = \frac{1}{7}$ $\quad$ $Pr(D_3) = \frac{1}{7}$ $\quad$ $Pr(D_4) = \frac{1}{7}$ $\quad$ $Pr(D_5) = \frac{1}{7}$ $\quad$ $Pr(D_6) = \frac{1}{7}$ $\quad$ $Pr(D_7) = \frac{1}{7}$

Information Gain Ratio

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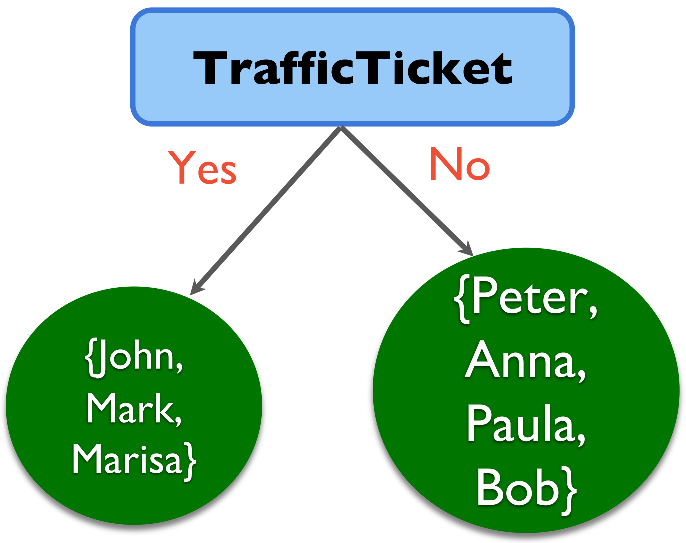
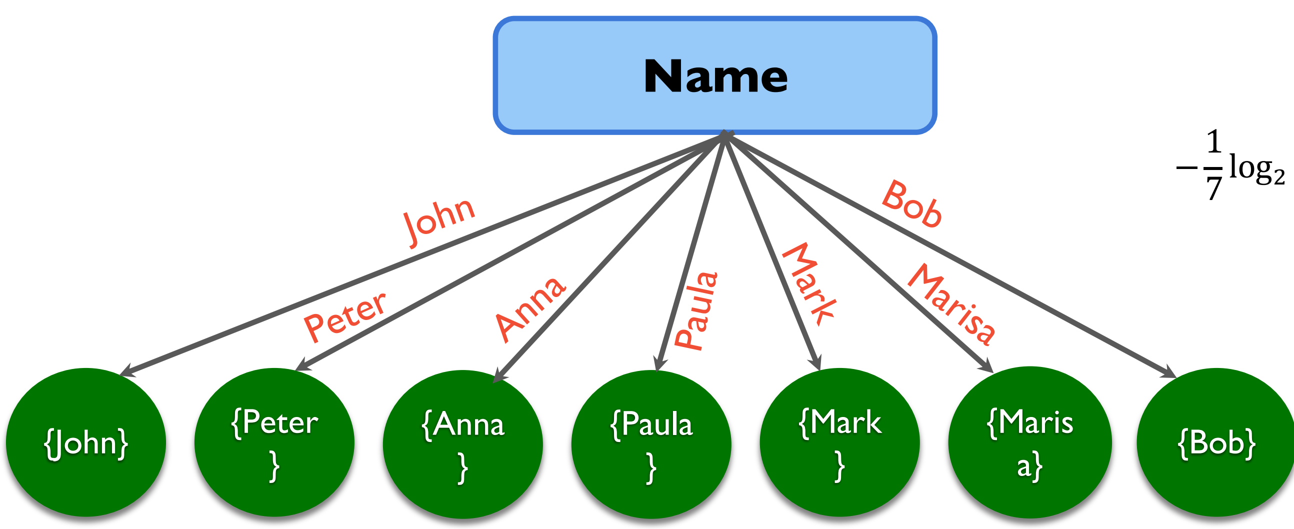
  - We've seen something like this before… Entropy! → which is called, in this context, **Split_Info**



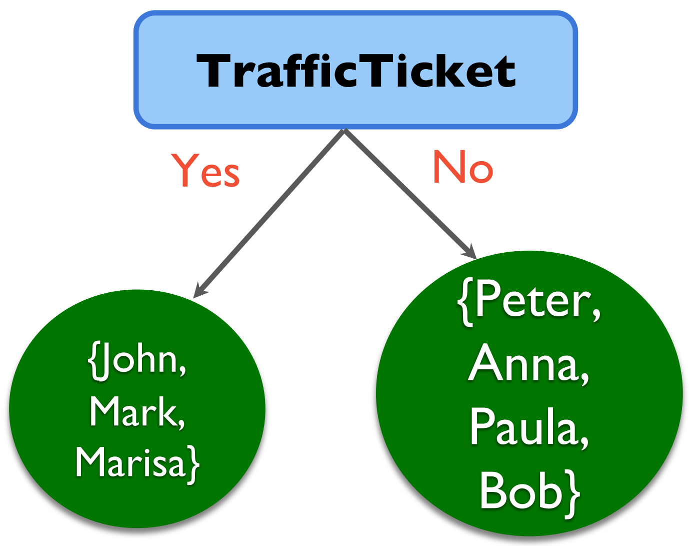
$$-\frac{3}{7}\log_2\left(\frac{3}{7}\right)-\frac{4}{7}\log_2\left(\frac{4}{7}\right)$$



$$-\frac{1}{7}\log_2\left(\frac{1}{7}\right)-\frac{1}{7}\log_2\left(\frac{1}{7}\right)-\frac{1}{7}\log_2\left(\frac{1}{7}\right)-\frac{1}{7}\log_2\left(\frac{1}{7}\right)-\frac{1}{7}\log_2\left(\frac{1}{7}\right)-\frac{1}{7}\log_2\left(\frac{1}{7}\right)-\frac{1}{7}\log_2\left(\frac{1}{7}\right)$$

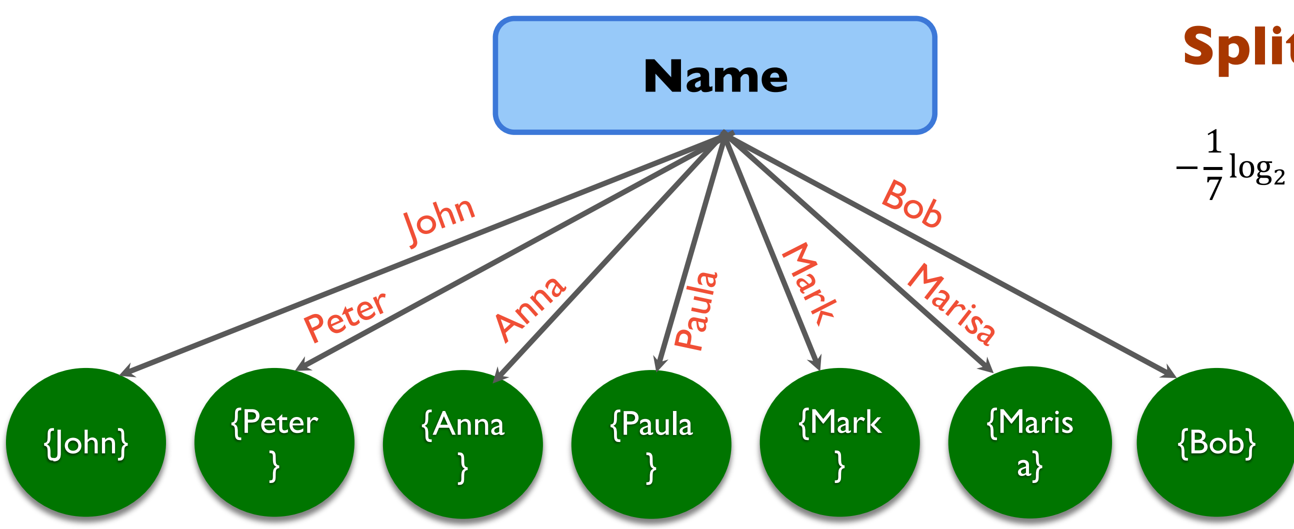
Information Gain Ratio

- How to quantify how “homogeneous" these quantities are?
  - We’ve seen something like this before… Entropy! → which is called, in this context, **Split_Info**



Split_Info(TrafficTicket) = $-\frac{3}{7}\log_2\left(\frac{3}{7}\right)-\frac{4}{7}\log_2\left(\frac{4}{7}\right)$

$= 0.98$



Split_Info(Name) =

$-\frac{1}{7}\log_2\left(\frac{1}{7}\right)-\frac{1}{7}\log_2\left(\frac{1}{7}\right)-\frac{1}{7}\log_2\left(\frac{1}{7}\right)-\frac{1}{7}\log_2\left(\frac{1}{7}\right)-\frac{1}{7}\log_2\left(\frac{1}{7}\right)-\frac{1}{7}\log_2\left(\frac{1}{7}\right)-\frac{1}{7}\log_2\left(\frac{1}{7}\right)$

$= 2.8$

Information Gain Ratio

- How to quantify how “homogeneous" these quantities are?
  - We’ve seen something like this before… Entropy! → which is called, in this context, **Split_Info**

Name	*Age*	*Gender*	*TrafficTicket*	Class: High-Risk Driver
John	43	M	Yes	**High Risk**
Peter	18	M	No	**Low Risk**
Anna	35	F	No	**Low Risk**
Paula	19	F	No	**Low Risk**
Mark	90	M	Yes	**High Risk**
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Split_Info(TrafficTicket) = $-\frac{3}{7}\log_2\left(\frac{3}{7}\right)-\frac{4}{7}\log_2\left(\frac{4}{7}\right) \quad = 0.98$

Split_Info(Name) = $-\frac{1}{7}\log_2\left(\frac{1}{7}\right)-\frac{1}{7}\log_2\left(\frac{1}{7}\right)-\frac{1}{7}\log_2\left(\frac{1}{7}\right)-\frac{1}{7}\log_2\left(\frac{1}{7}\right)-\frac{1}{7}\log_2\left(\frac{1}{7}\right)-\frac{1}{7}\log_2\left(\frac{1}{7}\right)-\frac{1}{7}\log_2\left(\frac{1}{7}\right) \quad = 2.8$

The larger value of Split_Info for Name suggests that this is a worse split than TrafficTicket

Information Gain Ratio

- The **Information Gain Ratio** combines two “measures" of how good a split (based on attribute A) is
  - Its **Information Gain**, as previously defined $\rightarrow$ $\text{Gain}_A(D)$ $\rightarrow$ higher is better
  - Its **Split_Info** $\rightarrow$ $\text{Split\_Info}(A)$ $\rightarrow$ higher is worse

$$\text{Gain\_Ratio}(A, D) = \frac{\text{Gain}_A(D)}{\text{Split\_Info}(A)}$$

Name	*Age*	*Gender*	*TrafficTicket*	Class: High-Risk Driver
John	43	M	Yes	**High Risk**
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Split_Info(TrafficTicket) = $-\frac{3}{7}\log_2\left(\frac{3}{7}\right) - \frac{4}{7}\log_2\left(\frac{4}{7}\right) = 0.98$

Split_Info(Name) = $-\frac{1}{7}\log_2\left(\frac{1}{7}\right) - \frac{1}{7}\log_2\left(\frac{1}{7}\right) - \frac{1}{7}\log_2\left(\frac{1}{7}\right) - \frac{1}{7}\log_2\left(\frac{1}{7}\right) - \frac{1}{7}\log_2\left(\frac{1}{7}\right) - \frac{1}{7}\log_2\left(\frac{1}{7}\right) - \frac{1}{7}\log_2\left(\frac{1}{7}\right) = 2.8$

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Marisa	19	F	Yes	**Low Risk**
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Gain$_{\textbf{TrafficTicket}}$(D) $= 0.466$

Split_Info(TrafficTicket) $= 0.98$

Gain$_{\textbf{Name}}$(D) $= 0.86$

Split_Info(Name) $= 2.8$

Information Gain Ratio

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Gain$_{\text{TrafficTicket}}$(D) $= 0.466$

Split_Info(TrafficTicket)$= 0.98$

Gain$_{\text{Name}}$(D) $= 0.86$

Split_Info(Name)$= 2.8$

In terms of **Information Gain** only, Name looks like **a good split**
However, its **Split_Info** suggests that Name it's **a bad split**

↓

Let's combine these into a single score that takes both into account
Gain_Ratio!

Information Gain Ratio

- The **Information Gain Ratio** combines two “measures" of how good a split (based on attribute A) is
  - Its **Information Gain**, as previously defined $\rightarrow$ $\text{Gain}_A(D)$ $\rightarrow$ higher is better
  - Its **Split_Info** $\rightarrow$ $\text{Split\_Info}(A)$ $\rightarrow$ higher is worse

$$\text{Gain\_Ratio}(A, D) = \frac{\text{Gain}_A(D)}{\text{Split\_Info}(A)}$$

In terms of **Information Gain** only, Name looks like **a good split**
However, its **Split_Info** suggests that Name it’s **a bad split**

Let’s combine these into a single score that takes both into account
Gain_Ratio!

$\textbf{Gain}_{\textbf{TrafficTicket}}\textbf{(D)} = 0.466$

$\textbf{Split\_Info(TrafficTicket)} = 0.98$

$$\text{Gain\_Ratio}(\text{TrafficTicket}, D) = \frac{0.466}{0.98} = 0.475$$

$\textbf{Gain}_{\textbf{Name}}\textbf{(D)} = 0.86$

$\textbf{Split\_Info(Name)} = 2.8$

$$\text{Gain\_Ratio}(\text{Name}, D) = \frac{0.86}{2.8} = 0.307$$

Information Gain Ratio

- The **Information Gain Ratio** combines two “measures" of how good a split (based on attribute A) is
  - Its **Information Gain**, as previously defined $\rightarrow$ $\text{Gain}_A(D)$ $\rightarrow$ higher is better
  - Its **Split_Info** $\rightarrow$ $\text{Split\_Info}(A)$ $\rightarrow$ higher is worse

$$\text{Gain\_Ratio}(A, D) = \frac{\text{Gain}_A(D)}{\text{Split\_Info}(A)}$$

In terms of **Information Gain** only, Name looks like **a good split**
However, its **Split_Info** suggests that Name it’s **a bad split**

Let’s combine these into a single score that takes both into account
Gain_Ratio!

$$\text{Gain\_Ratio}(\text{TrafficTicket}, D) = \frac{0.466}{0.98} = 0.475$$

This criterion “understands" that splitting based on TrafficTicket is better than splitting based on Name

$$\text{Gain\_Ratio}(\text{Name}, D) = \frac{0.86}{2.8} = 0.307$$

Criteria for Selecting an Attribute to Test

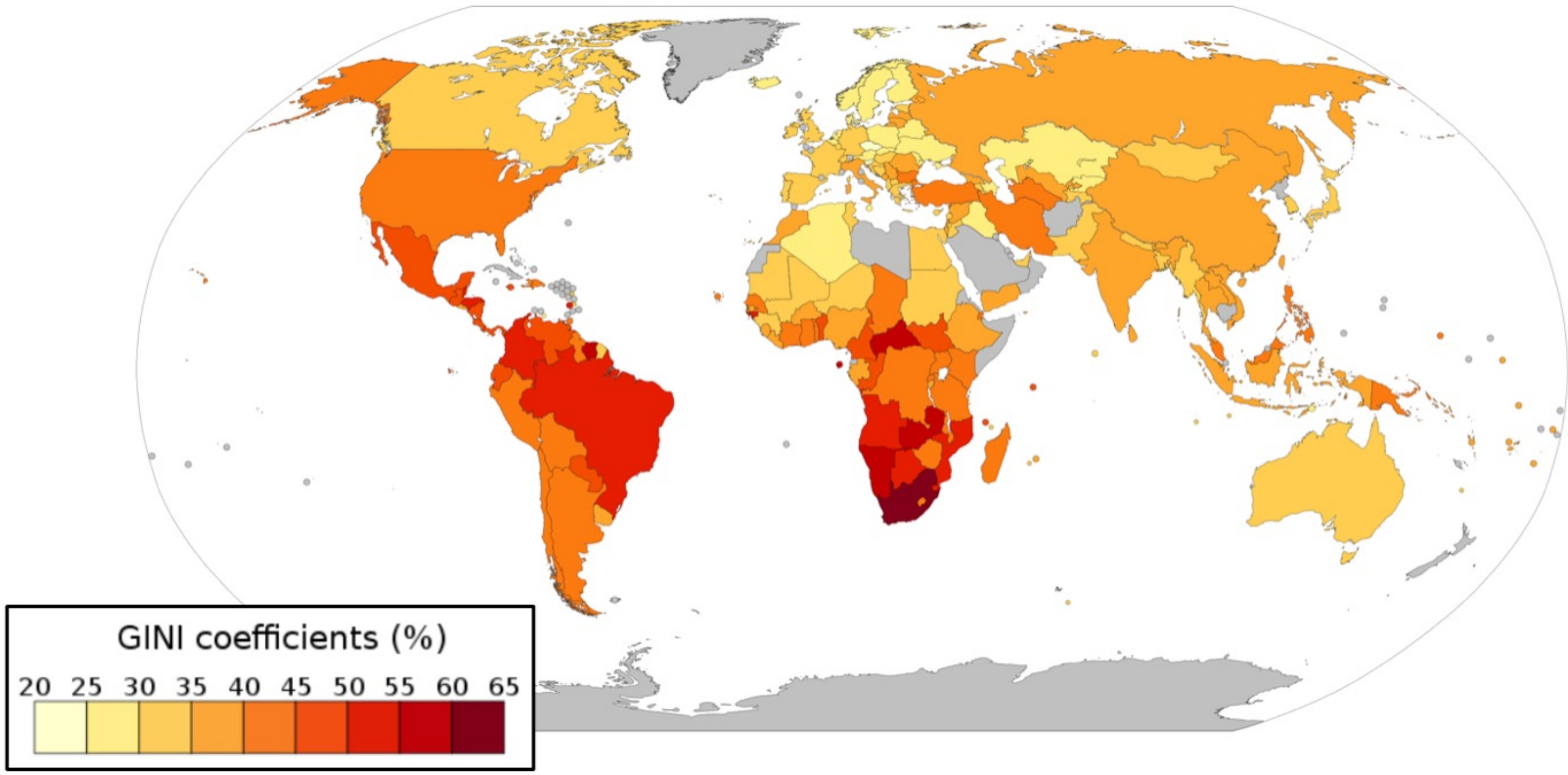
- We have discussed one possible criterion for selecting which attribute to test
  - **Information Gain**

- Many other criteria have been proposed — each with different properties
- Intuitively:
  - A split that keeps the same proportion of classes in each partition is **useless**
  - A split where the instances in each partition have the same class is **useful**!

- Main criteria for selecting which attribute to test:
  - **Information Gain** - ID3 Algorithm (Quilan, 1987)
  - **Information Gain Ratio** - C4.5 Algorithm (Quilan, 1988)
  - **Gini Impurity** - CART Algorithm (Breiman, 1984)

Gini Criterion

- Originally proposed to quantify how uneven income is across a population

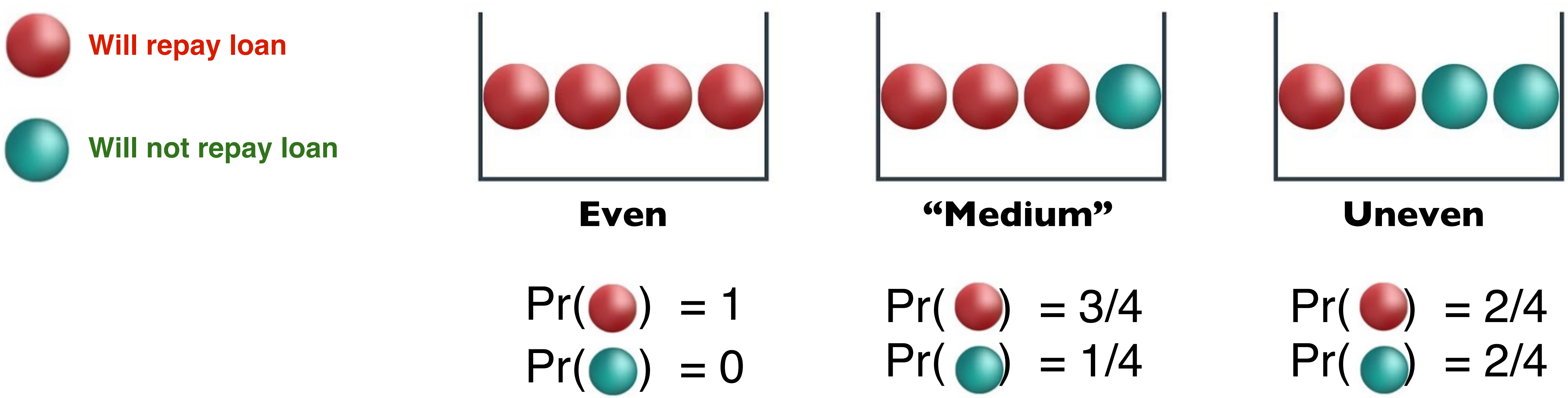


- Gini coefficient → how uneven income/wealth distribution across a population is
  - Gini = 1 → very uneven income/wealth distribution across a population
  - Gini = 0 → very even income/wealth distribution across a population

Gini Criterion

- Gini coefficient → how uneven income/wealth distribution across a population is
- In the context of decision trees
  - how uneven (or non-homogeneous) are the classes after a split

Let's suppose we test Age, and the instances associated with Age=Young look like this



Gini Criterion

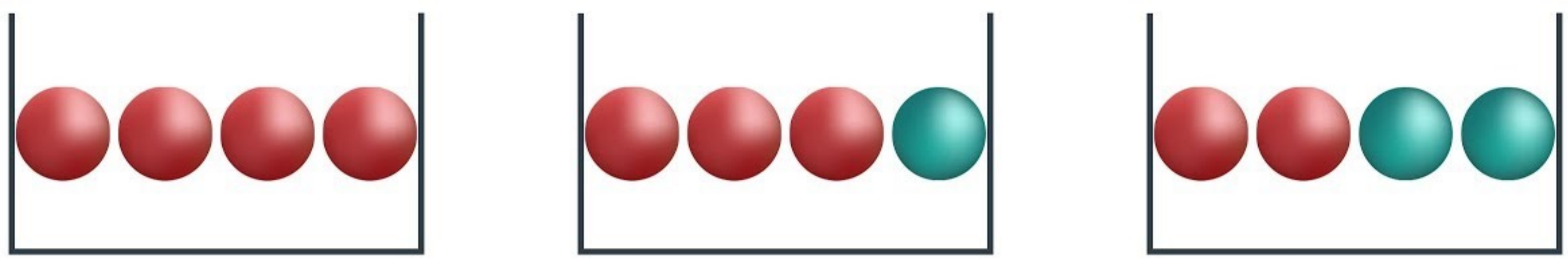
- Gini coefficient → how uneven income/wealth distribution across a population is
- In the context of decision trees
    - how uneven (or non-homogeneous) are the classes after a split

Gini(D) = 1 - (Pr( )^2 + Pr( )^2)

Let's suppose we test Age, and the instances associated with Age=Young look like this

Will repay loan

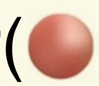
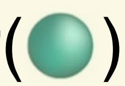
Will not repay loan



	Even	"Medium"	Uneven
	Pr( ) = 1	Pr( ) = 3/4	Pr( ) = 2/4
	Pr( ) = 0	Pr( ) = 1/4	Pr( ) = 2/4
Gini ⇒	$1-(1^2+0^2)$ $=0$	$1-((3/4)^2+(1/4)^2)$ $=0.375$	$1-((2/4)^2+(2/4)^2)$ $=0.5$

Gini Criterion

- Gini coefficient → how uneven income/wealth distribution across a population is
- In the context of decision trees
  - how uneven (or non-homogeneous) are the classes after a split

Gini(D) = 1 - (Pr()^2 + Pr()^2)

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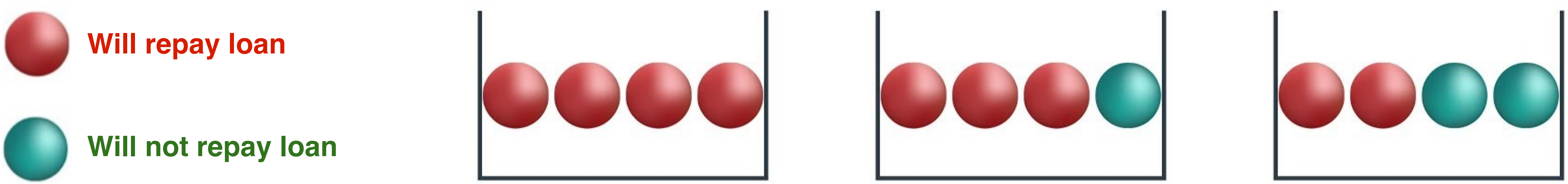


Even | **"Medium"** | **Uneven**

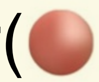
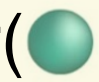
Gini ⇒

Even	"Medium"	Uneven
$1-(1^2+0^2)$ $=0$	$1-((3/4)^2+(1/4)^2)$ $=0.375$	$1-((2/4)^2+(2/4)^2)$ $=0.5$

more homogenous partition → ideal result of a split
(smaller value of the Gini coefficient)

Gini Criterion

- In the context of decision trees
  - how uneven (or non-homogeneous) are the classes after a split

Gini(D) = 1 - (Pr()^2 + Pr()^2)

- More generally, if there are m classes in a dataset D

$$\text{Gini}(D) = 1 - \left(\sum_{i=1}^{m} (p_i)^2\right)$$

where p_i be the probability that the label/class i occurs in instances in a dataset D

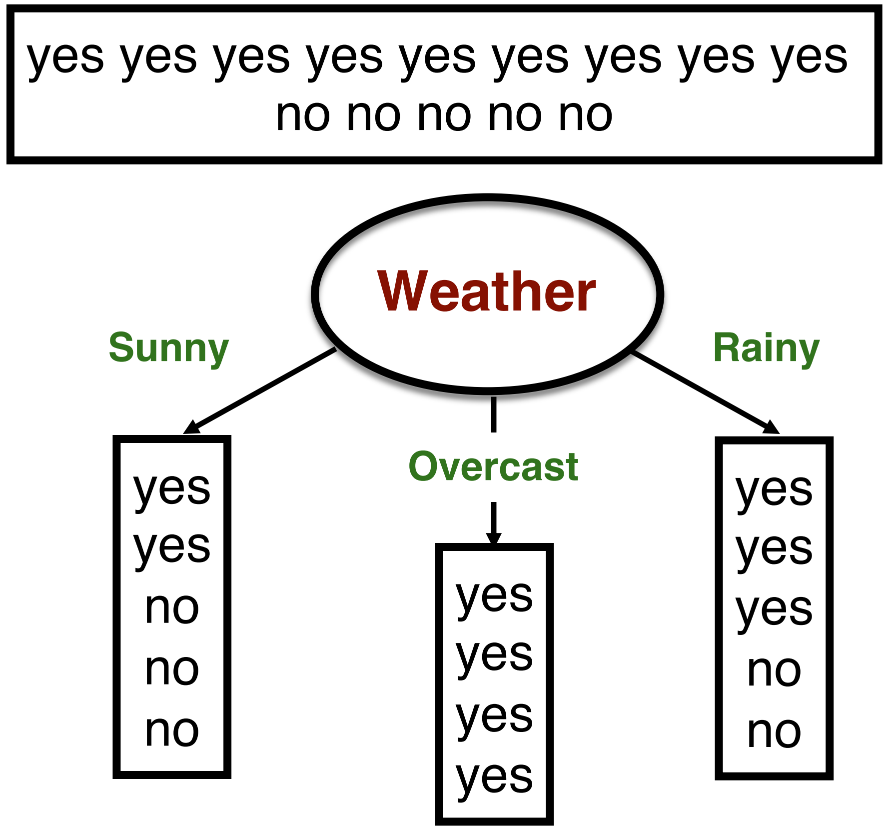
Gini Criterion

- Decision tree to predict whether a person will play tennis

Weather	Temperature	Humidity	Windy	PlayTennis
Sunny	Hot	High	False	No
Sunny	Hot	High	True	No
Overcast	Hot	High	False	Yes
Rainy	Mild	High	False	Yes
Rainy	Cool	Normal	False	Yes
Rainy	Cool	Normal	True	No
Overcast	Cool	Normal	True	Yes
Sunny	Mild	High	False	No
Sunny	Cool	Normal	False	Yes
Rainy	Mild	Normal	False	Yes
Sunny	Mild	Normal	True	Yes
Overcast	Mild	High	True	Yes
Overcast	Hot	Normal	False	Yes
Rainy	Mild	High	True	No

- Let's consider testing Weather

Original dataset: 9 instances "Yes"
5 instances "No"



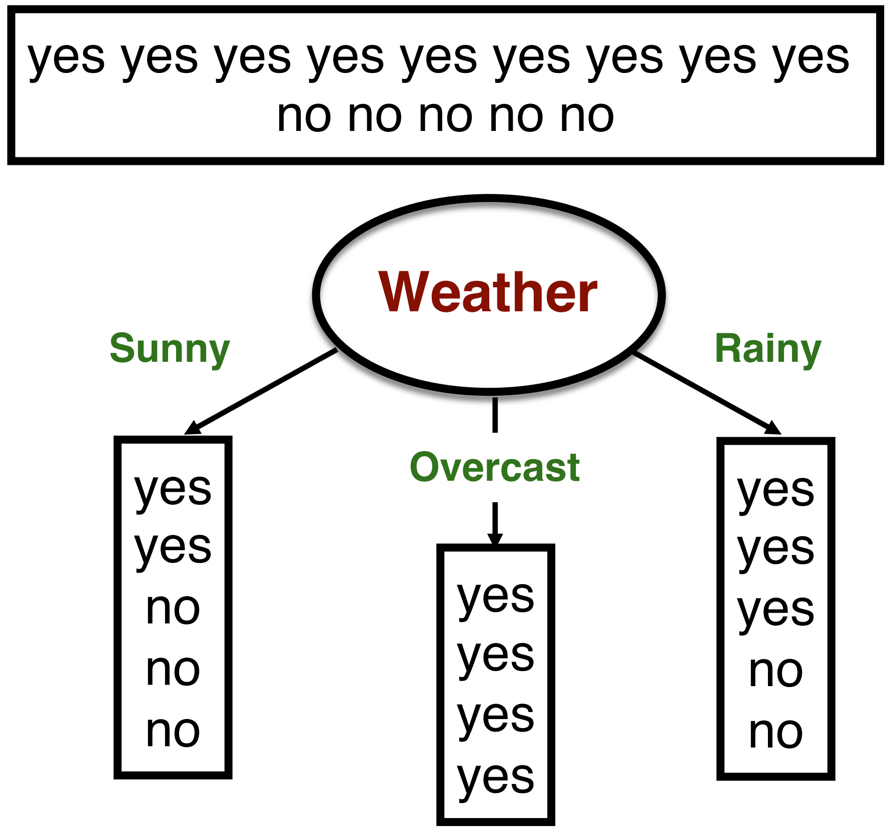
Gini Criterion

- Decision tree to predict whether a person will play tennis

- **Gini coefficient of the original dataset:**
  - **Gini**(9/14, 5/14) = 1 - ($(9/14)^2 + (5/14)^2$)
    = **0.459**
- **Gini coeff. of partitions resulting from testing Weather:**
  - **Weather=Sunny**
    - **Gini**$_{\text{Sunny}}$(2/5, 3/5) = 1 - ($(2/5)^2 + (3/5)^2$)
      = 0.48
  - **Weather=Overcast**
    - **Gini**$_{\text{Overcast}}$(4/4, 0/4) = 1 - ($(4/4)^2 + (0/4)^2$)
      = 0
  - **Weather=Rainy**
    - **Gini**$_{\text{Rainy}}$(3/5, 2/5) = 1 - ($(3/5)^2 + (2/5)^2$)
      = 0.48
- **Average Gini coefficient of the resulting partitions**
  - (5/14)x0.48 + (4/14)x0 + (5/14)x0.48 = **0.3428**

- Let's consider testing Weather

Original dataset: 9 instances "Yes"
5 instances "No"



Gini Criterion

- Decision tree to predict whether a person will play tennis

- **Gini coefficient of the original dataset:**
  - **Gini**(9/14, 5/14) = 1 - ($(9/14)^2 + (5/14)^2$)
    = **0.459**
- **Gini coeff. of partitions resulting from testing Weather:**
  - **Weather=Sunny**
    - **Gini**$_{Sunny}$(2/5, 3/5) = 1 - ($(2/5)^2 + (3/5)^2$)
      = 0.48
  - **Weather=Overcast**
    - **Gini**$_{Overcast}$(4/4, 0/4) = 1 - ($(4/4)^2 + (0/4)^2$)
      = 0
  - **Weather=Rainy**
    - **Gini**$_{Rainy}$(3/5, 2/5) = 1 - ($(3/5)^2 + (2/5)^2$)
      = 0.48
- **Average Gini coefficient of the resulting partitions**
  - (5/14)x0.48 + (4/14)x0 + (5/14)x0.48 = **0.3428**

Testing the attribute **Weather**:
Gini(Weather) = **0.3428**

- Now proceed similarly as when selecting attributes via Information Gain…

- Compute Gini coefficient of each candidate attribute
- Split dataset using the attribute with the **lowest Gini coefficient**

Gini Criterion

Formally:

- Let p_i be the probability that the label i occurs in instances in a dataset D

- $\text{Gini}(D) = 1 - \left(\sum_{i=1}^{m} (p_i)^2\right)$ is the Gini coefficient of an arbitrary dataset D (m is the number of classes/labels)

- Assume that the attribute A can take up v values

  (that is, if we split D based on attribute A, we will end up with v partitions)

- Let $\text{Gini}_A(D) = \sum_{j=1}^{v} \frac{|D_j|}{|D|} \text{Gini}(D_j)$ be the Gini coefficient associated with splitting D based on A

- At each step, the algorithm splits the instances based on the attribute A with **lowest Gini coefficient**

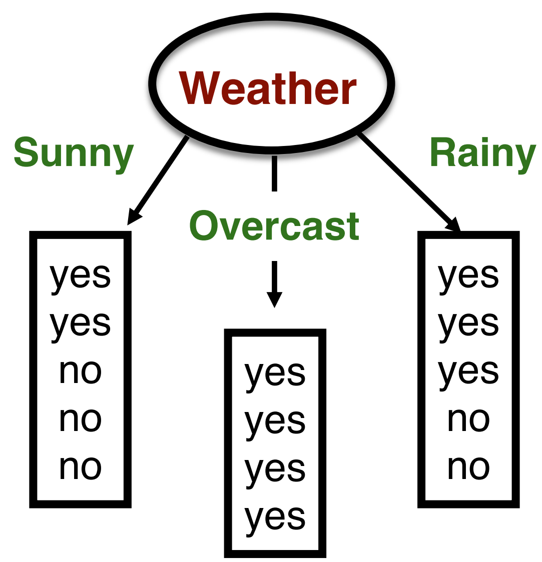
Criteria for Selecting an Attribute to Test

- Main criteria for selecting which attribute to test:
  - **Information Gain** - ID3 Algorithm (Quilan, 1987)
  - **Information Gain Ratio** - C4.5 Algorithm (Quilan, 1988)
  - **Gini Impurity** - CART Algorithm (Breiman, 1984)

- **Empirically:**
  - Information Gain Ratio is almost always better than Information Gain
    - in terms of predictive power and complexity of the resulting decision trees
  - However, in practice
    - which criterion will work best depends heavily on the application
    - should test them all and compare the resulting performances

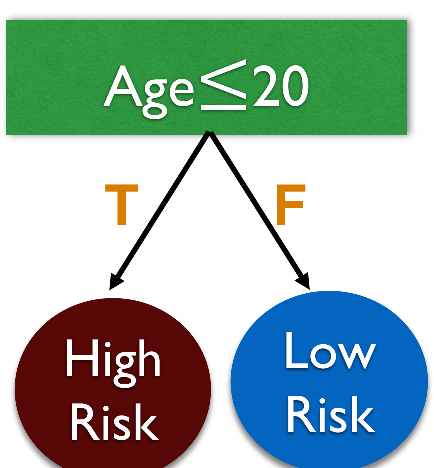
Dealing with Numerical Attributes

- So far we have studied how to select which **categorical attribute** to split



One branch per possible value of the attribute

- How do we decide a splitting point/value in case of **numerical attributes**?



Consider deciding how to split the attribute **Age**

Pick a threshold value, V

Generate two branches/disjoint partitions:

- one partition with instances s.t. **Age** $\leq V$
- one partition with instances s.t. **Age** $> V$

Dealing with Numerical Attributes

- How do we decide a splitting point/value in case of **numerical attributes**?

  - one partition with instances s.t. **Age** $\leq V$
  - one partition with instances s.t. **Age** $> V$

1) Sort the instances according to the value of the attribute

Name	***Age***	***Gender***	***TrafficTicket***	**Class: High-Risk Driver**
John	43	M	Yes	**High Risk**
Peter	18	M	No	**High Risk**
Anna	35	F	No	**Low Risk**
Paula	19	F	No	**High Risk**
Mark	90	M	Yes	**High Risk**
Marisa	21	F	Yes	**High Risk**
Bob	30	M	No	**Low Risk**

Dealing with Numerical Attributes

- How do we decide a splitting point/value in case of **numerical attributes**?

  - one partition with instances s.t. **Age** $\leq V$
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Dealing with Numerical Attributes

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  - one partition with instances s.t. **Age** $\leq V$
- one partition with instances s.t. **Age** $> V$

1) Sort the instances according to the value of the attribute

2) Evaluate splits done using as threshold the mean values between consecutive Ages

Name	***Age***	***Gender***	***TrafficTicket***	**Class: High-Risk Driver**
Peter	18	M	No	**High Risk**
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Age $\leq$ 18.5

Dealing with Numerical Attributes

- How do we decide a splitting point/value in case of **numerical attributes**?

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Anna	35	F	No	Low Risk
John	43	M	Yes	High Risk
Mark	90	M	Yes	High Risk

Age≤18.5

Age≤20

Dealing with Numerical Attributes

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John	43	M	Yes	**High Risk**
Mark	90	M	Yes	**High Risk**

Age≤18.5

Age≤20

Age≤25.5

Dealing with Numerical Attributes

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Anna	35	F	No	Low Risk
John	43	M	Yes	High Risk
Mark	90	M	Yes	High Risk

- Age$\leq$18.5
- Age$\leq$20
- Age$\leq$25.5
- Age$\leq$32.5

Dealing with Numerical Attributes

- How do we decide a splitting point/value in case of **numerical attributes**?

  - one partition with instances s.t. **Age** $\leq V$
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Mark	90	M	Yes	High Risk

Age≤18.5

Age≤20

Age≤25.5

Age≤32.5

Age≤39

Dealing with Numerical Attributes

- How do we decide a splitting point/value in case of **numerical attributes**?

  - one partition with instances s.t. **Age** $\leq V$
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Bob	30	M	No	Low Risk
Anna	35	F	No	Low Risk
John	43	M	Yes	High Risk
Mark	90	M	Yes	High Risk

Age≤18.5

Age≤20

Age≤25.5

Age≤32.5

Age≤39

Age≤66.5

Dealing with Numerical Attributes

- How do we decide a splitting point/value in case of **numerical attributes**?

  - one partition with instances s.t. **Age** $\leq V$
  - one partition with instances s.t. **Age** $> V$

  1) Sort the instances according to the value of the attribute

  2) Evaluate splits done using as threshold the mean values between consecutive Ages

Age≤18.5	Age≤20	Age≤25.5	Age≤32.5	Age≤39	Age≤66.5

  3) Pick the split threshold that maximizes the criterion of interest ***(Info. Gain, Gini, etc.)***

- It has been shown that, for most commonly-used splitting criteria
  - testing only thresholds that correspond to such mean values is sufficient

Decision Trees: Pros and Cons

- **Pros:**
  - Simple for humans to understand and interpret
  - Handles both numerical and categorical attributes
  - Requires little data preparation *(e.g., no need to normalize attributes)*
  - Performs well with large datasets
  - “Automatically" ignores irrelevant attributes not useful to predict the class/label

- **Cons:**
  - Non-robust: small variations in the dataset can generate completely different trees
  - Often generate overly-complicated trees that overfit to training data
    - i.e., that do not generalize well (make correct predictions) to new instances
  - Although it is possible to deal with numerical attributes, it is time-consuming
    - estimates suggest that processing them takes ~70% of execution time (Catlett, 1991)